APPENDIX F

String Lining of Curves Made Easy*

A Method of Correcting Alinement Which Is Quicker and More Convenient Than Use of Engineering Instruments

By CHARLES H. BARTLETT

This is the first of a series of articles describing a new method for relining curves by means of a string in place of a transit. In view of the increasing importance of accurate curve maintenance and the necessity for checking curve alinement frequently, it is believed that this method will be of interest and help to division engineers; roadmasters and foremen. It has been adopted by a large western road after extensive trial and is employed in the maintenance of several thousand miles of this line. Each article will be complete in its description of the particular portion of the method, while the series as a whole will constitute a manual of instruction to which anyone using the method may refer as much as he finds necessary. The author developed this system when an instrumentman in the office of the division roadmaster of the St. Louis division of the Illinois Central.—EDITOR.

IT is a well known fact that all railroad curves are made up of circular curves and easements or spirals. The circular curves are staked out by engineers before the track is built; while the easements are more usually installed after the curve is laid. The above statement is especially true of curves which were laid out many years ago, when the use of an easement curve was considered an unnecessary refinement of engineering. It is an equally well known fact that no curve, no matter how well it may be ballasted or how carefully it may be maintained, will remain as it was originally staked out. This change, due to many different forces (such as temperature changes, continual pounding by passing trains, shifting of the roadbed, etc.), produces what are known as "sharp" and "flat" spots in the original curve. By a sharp spot is meant a portion of the curve whose curvature is greater or "sharper" than that of neighboring portions of the curve, or, less commonly, than the curvature as first staked out. Similarly, a flat spot is one whose curvature is less than that of the adjacent portions of the same curve. It is obvious that such a condition is not desirable, and equally obvious that the sooner it is corrected, the less discomfort will be caused to passengers and the safer the track will be for all trains, passenger and freight. It is the purpose of string lining to correct the defects of alinement and to give to the curve that uniformity which insures both good riding qualities and safety, restoring the curve to its original shape or nearly so.

Although string lining, in one form or another, has been in use on some of the railroads of this country for many years, its usehas not been widespread and many roads have preferred to leave the work of realigning curves to the engineering department, or else entirely to the track foremen, each of whom has had his individual way of accomplishing the work.

This practice has resulted in a condition found on almost every road in the country, whereby on one section all curves will be well lined and properly elevated, while on the next section they will be just the opposite—the reason, of course, being the difference in the ability of the two section foremen to line their curves. Such a condition could be remedied quite easily if a standard procedure for curve lining could be adopted. Natually it follows that any system adopted as standard must be easily learned and easily remembered by all foremen and supervisors; and, moreover, that such a system must reduce to a minimum not only the time required for calculations, but also the effect of judgment or experience. In other words, it must be nearly "mechanical" in its application. It is claimed for the system described in this series of articles that it possesses to a high degree all these qualifications. The proof will, of course, rest in the application of the principles to actual conditions.

It should be pointed out at this place that this system is not that commonly known as "cording a curve." The latter method consists of measuring the middle ordinates to a curve from a cord stretched between points on the curve, averaging what appears to be the predominating ordinate, and then throwing the track, by means of lining bars or similar devices, back and forth until it is approximately uniform. While this method will undoubtedly work, after a fashion, any foreman who has ever tried it knows that he will have to go over the curve several times before a satisfactory result can be obtained, and the labor required and the time involved must surely have struck him as so much wasted effort. Such a man will readily concede that if the proper throw could be made the first time much needless work would be saved. The proposed method points out exactly what correction is needed at each point in order to make a true curve.

Another point which should be emphasized here is that no eleborate equipment is necessary. No piano wire and no delicate measuring instruments are required. All that is needed is a good stout piece of cord, similar to that used by bricklayers or masons, and an ordinary ruler whose divisions start at the end of the rule in order to allow it to be placed against the gage line of the high rail, as explained later. Moreover, there are no complicated mathematical expressions to use, as the process consists in the addition and subtraction of small numbers representing the middle ordinates as measured. Frequent checks on the work can be made, as explained later, so that if a mistake is made it will be detected quickly, before the work has gone too far.

Finally, the system *works*. This has been demonstrated in actual practice by re-alining nearly a thousand curves by its use. And it works, not only from the standpoint of the maintenance man, by giving him better track in less time and with less expenditure of labor and money, but also from the standpoint of the engineer, inasmuch as by its use can be determined all the properties of a curve which can be ascertained by using an instrument. That is, the central angle between tangents, the central angles of all spirals, the points of spiral, points of curve, degree of curvature, rate of change of curvature of spiral, etc.—all can be computed readily and easily within satisfactory limits of error. When to this fact are added the many other advantages—such as the fact that the system is from five to ten times as rapid as transit lining, the engineer as well as the foreman will realize that it will pay him to investigate the possibilities of this method of string lining.

The Method Is Rapid

The time required to master the fundamental principles of the entire method varies from two or three hours to about eight or nine hours. One day's working time has always been found sufficient to enable a new man to obtain a thorough understanding of all the rules and necessary operations.

The various operations in setting stakes for a curve, such as taking the original data, figuring the curve, and setting the stakes, have all been carefully timed. It takes, on an average, about 1/4 minute per station to take the original ordinates of a curve. This is an average figure, and takes into account delays due to waiting for trains to pass, walking between stations, rechecking any doubtful ordinates, and finding the point of ending and beginning of the curve (by trial method to be outlined).

It takes, on an average, about 1-1/2 minutes to distribute stakes at each string lining station (from the original bundles), drive an iron pin into rock ballast, shake the pin to make a good hole, drive a wooden stake, and set the tack in the stake. In cinder ballast the average time per stake set in the field is about 1 to 1-1/4 minutes, depending upon the firmness and density of the ballast. This makes the total time required to perform all the field operations in lining a curve equal to about 1-3/4 minutes per stake in rock ballast, and 1-1/4 to 1-1/2 minutes per stake in cinder ballast.

Several Trials Are Required

It is obvious that in attempting to line a curve which is badly out of shape, several trials will be required before the best curve can be found. If a foreman and his gang of men set out to reline a curve by eye only, several attempts will be required before a satisfactory curve can be obtained; and the same is true of the computations required by the system described in this series of articles, the only difference being that instead of the actual track on the curve beng changed, only the figures representing the curve are changed. It has been found, consequently, that an ordinary school slate, ruled into columns (as described later), furnishes the easiest and most satisfactory way of making the changes required, inasmuch as it requires more time to make erasures from a sheet of paper. The length of time required to make the computations on the slate will, of course, depend upon the individual and upon the number of curves he has lined. Curves requiring no more than a foot throw either in or out, can ordinarily be lined upon a slate at the rate of from 1 to 2 stations a minute. The author has lined (on the slate) curves of 86 to 90 rails in 20 and 30 min. time, whereas some harder and shorter curves which were badly out of line have taken as much as three or four hours, in order to obtain a curve which would not throw beyond a certain fixed limit. The figure of 1 to 2 stations a minute includes the entire time required to line the curve on the slate, from the time of starting until the curve is fully lined; it includes making all changes, erasures, etc.

Thus, according to the above average figures, a curve in rock ballast, a mile long, would require about seven hours to re-line, from the time of starting in to get the original data until the last stake was set. As a matter of record, several curves a mile long have been lined by this method at the rate of six hours per mile. All of these curves were compound curves (before lining), and were spiraled. In relining them by the string system, proper spirals were placed at each end and between the branches of the compounds. The author's experience in lining curves with a transit has led him to believe that the time it would require to line a compound curve (with two branches), spiral both ends and insert a spiral between the branches of the compound, and drive a stake every 33 ft. in rock ballast, would be anywhere from three to five working days of eight hours each. On such a curve (a mile long), the intersection angle could not be run, in all probability, and the matter of lining an entire mile without "running off the embankment" would be quite a problem.

In addition to the fact that a curve can be figured and the stakes set much more quickly by the string lining method than by the transit method, there are other advantages which make for a considerable saving in time. For example, passing trains and motor cars do not cause any serious loss of time. With an instrument, every time a train or motor car goes by, a new set-up must be made, which takes from three to eight or ten minutes, depending upon how far the flagman has to go for a second backsight. With the string lining method, the only time lost is that which it takes the train or motor car to pass the working point. This is an advantage of considerble weight on heavy traffic lines.

String lining is cheaper than transit lining because of the time save, because fewer mistakes are made, because a better curve is obtained, and because the throws are kept to a minimum. In transit lining, a party must necessarily consist of an instrumentman, a rodman and chainman. The combined salaries of these men will equal about two to three times that of a chainman and two section laborers for the same length of time. Adding to this fact the saving in time effected, we have, considering string lining as say four times as rapid as transit lining, the cost in engineering labor to use the string method as approximately one-eighth of that with the transit method. These figures, if anything, are conservative. The actual saving will run as high as 9/10 in most cases. Not only are the salaries of the instrumentman and rodman saved on each curve, but these two men are than available for other work in the engineering department.

Facilitates Compounding

Further, string lining permits compounding a revised curve slightly for a given distance, usually quite short, in such a manner that the saving in throw effected is marked. This is a valuable property for the maintenance man, since it enables him to install a smooth-riding curve with a minimum of throw, and consequently, with a minimum of time and labor. The time of a section gang can be cut down by several hours in this way, thus realizing a big saving. Suppose that at string lining station number 10 of a curve, we change a revised ordinate by 1 (tenth of an inch) in order to decrease the throws, then as will be explained later, at the end at some such station as 67, say, we have effected a saving in throw of twice 67-10 or 114 tenths of an inch (11.4 in.); yet we have compounded the curve only 5-1/4 minutes and for a length of only one rail. It would never have occurred to an instrumentman with a transit to make such a compound, and he would have carried his curve through to the end, with the consequent big throws. String lining, then, permits of keeping the track close on the old bed (a big advantage) and yet obtaining a smooth-riding curve.

By making it easier for the trackman to line his curve (by placing the stakes every 33 ft. instead of every 50 or 100 ft.), we avoid the small sharp and flat spots which creep in because of the fact that a foreman has to line between the 100 ft. transit stakes by his eye. We accordingly prevent the curve from getting so quickly into bad line once more, and thus effect another saving in maintenance cost.

Work Can Be Inspected in Advance

Since a record of the throws required at each joint or string lining station to line a curve into a new and better shape is obtained from the slate or calculation paper, the operator has a record, in black and white, of how much he proposes to throw the curve. He can take this record to the roadmaster or assistant engineer, as the case may be, to look over at his leisure and inspect the changes necessary.

If the work were to be done with a transit, the instrumentman would first have to set spikes or stakes, and then go to every transit station and measure the throw, if such a record were to be given his superior officers before the curve was actually lined in the field. The advantage of having such a record as string lining gives is readily understood by anyone who handles such matters on a railroad. The size of the gang required to do the work of lining, the approximate amount of time required to do the work, and all other arrangements or information relative to the task of re-alining the curve are known and determined before a start is made.

String lining permits of so compounding a curve (if necessary, of course) that the throws at or near a permanent track structure can be made either zero or negligible. This also is a means of avoiding a great deal of unnecessary work and of saving time and expense. Such small throws as are required can be made upon the slate, so that the entire amount of shifting of such obstacles can been seen by the man in change of moving them before the curve is lined. To do this with a transit would require a great deal of time and the running in of many trial curves before a final curve is selected.

The equipment necessary to do string lining is much simpler and much cheaper than that required with a transit. In addition to this, it is easier to carry around, no care need be felt for its safety or adjustment; and, finally, expensive errors in instrument work done by more or less inexperienced men are entirely avoided.

Summary

The principal advantages of string lining, then, are as follows:

(1) It is much *cheaper* than any other method.

(2) It is much *quicker* than any other method.

(3) It is much *easier* than any other method.

(4) It is *easily learned, easily remembered,* and *easy to use.*

(5) *Errors are* readily *detected by several checks on the work.*

(6) *Throws can be governed almost at will.*

(7) Spirals are no longer an "affliction" to install, but are so easy, and help to decrease the throws required to line a curve by so great an amount that their installation becomes easy and automatic.

(8) A record of proposed changes or throws is obtained and is ready for inspection before any work is done.

(9) It lowers maintenance costs.

(10) It permits of relining curves annually at small expense of time and labor.

(11) It gives a more satisfactory curve.

(12) No expensive or easily breakable equipment is required.

(13) Less men are needed to do the work of figuring the curve.

How to Measure a Curve

A Description of the Steps to Be Taken in Securing Data Regarding an Existing Curve

THE WORK of string lining a curve falls into three natural subdivisions. These are, in the order of their occurrence:

1. The taking of the necessary field data of the existing curve, and their arrangement and recording in such manner that the necessary computations will be shortened and facilitated as much as possible, in order that mistakes will be rendered improbable.
2. The computation of the throws required to correct the various defects of alinement of the existing curve.
3. From the corrections computed, the setting of stakes or other guides by means of which the section foreman and his gang will be enabled to re-aline the curve.

Of course, a fourth subdivision might be added—namely, the checking of the revised curve after the work of re-alining has nominally been completed. However, this is practically never done and, as a matter of fact, is needless if the work has been carefully done throughout.

This article deals with the methods of taking the required information—that is, the characteristics—of the existing curve. Before discussing these methods it seems advisable to recall certain properties of the circular curve.

Basis of String Lining

Railway curves can be divided into two broad classes: (1) circular curves and (2) spiral or easement curves. In the original location of a line, before the track is laid, the circular curve only is staked out; the spiral is left until the greater portion of the curve is actually laid and is then staked out by the engineer or is left to the track foreman to put in by eye. For this reason, and also because the spiral requires special treatment, the circular curve will be considered first.

Both theory and experience prove that, in order to provide and maintain good riding qualities in a curve, it is necessary to keep truly circular that portion which was originally constructed as a circle. This can be done in several ways. With a transit, advantage is taken of the principle that, for equal chord lengths along a circle, the deflections from a tangent at the point of curve are (angularly) equal. String lining takes advantage of the principle that, for equal chord lengths along a circle, the ordinates from the center point of the chord, measured perpendicularly thereto, are equal. And, just as for a given degree of curve there is a characteristic deflection for each chord length, so for that same degree of curve and same chord length there is a single characteristic middle ordinate. Therefore, if the middle ordinates of a given curve, measured from equal chord lengths, are all equal, that curve is a true circle.

It is well known that a curve which is a true circle when originally laid will eventually, as a result of outside forces such as engine and car wheels, temperature changes, etc., become a series of curves, some or all of which may be of different degrees of curvature. This naturally makes for rough riding track. It is the purpose of string lining to restore the circular character of the curve by making the unequal middle ordinates of the actual curve all equal. In order to do this with the minimum of expense, time and interruption of traffic, it is first necessary to obtain a record of the middle ordinates, for a given chord length, on the entire curve.

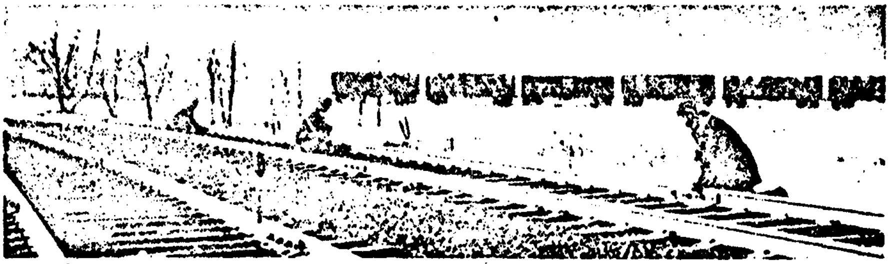
Selecting the Chord Lengths

The best length of chord depends on certain factors, chief of which is the length of rails in the track. If the entire curve is composed of rails of equal length, such as 30 or 33 ft., the best chord length is equal to twice the length of rail; that is, 60 or 66 ft. Since all ordinates are measured from the chord to the high rail of the curve (because measurements can then be taken to the gage line of the rail) only the length of rails in the outer rail need be considered. If, however, the rails vary in length (as sometimes happens) the writer recommends a chord length of 66 ft., because all the tables used by him are based on the use of that length of chord and, therefore, such data as may be required can be obtained directly from tables, without computation.

The reason for selecting a chord equal in length to two rails is that this permits of setting a stake at every joint, if desired. The advantages of a center stake every rail length, at the joint, over a center stake every 50 or 100 ft., are readily realized, and are fully appreciated by the track foremen who have to do the work of lining. A much more uniform curve is secured with the expenditure of no more time or labor. Only the very best of foremen are able to line 100 ft. of curve of any degree without having

*This is the second of a series of articles on the string lining of curves, describing the manner in which the line can be corrected by trackmen without the use of instruments other than a piece of string and an ordinary rule. The first article of this series, which appeared in the January issue, page 4, presented the merits of this practice in contrast with the use of a transit. The third article will appear in the April issue.

for String Lining*†



slight sharp and flat spots. These irregularities, although almost unnoticeable at first, lead very quickly to generally poor alinement.

The accompanying table gives the middle ordinate in tenths of an inch, for a one-degree curve for a number of chord lengths.

Measuring the Curve

Theoretically, on perfectly straight track, the middle ordinate (for any length of chord) is zero. If the actual tangent track was perfectly straight, the first ordinate with which we should have to deal would be a zero, at the end of the curve. However, for reasons explained later, it is frequently advisable to start well back on the straight track—say five or six rail lengths—to measure the middle ordinates. Although string lining does not undertake to change tangent alinements, it does permit of making detailed corrections thereto at the ends of curves, if there are small variations in the straight track.

For example, it will nearly always be found that the last ordinates—the first and last on the curve, and the next adjoining ones just off the curve—are not zero. A typical case, taken from an actual curve, measured with a 66-ft. chord is as follows:

Joint Number	Middle Ordinate
Joint Number 1	0.2 in.
" " 2	0.2 "
" " 3	0.0 "
" " 4	0.1 "
" " 5	0.2 "
" " 6	0.0 "
" " 7	0.4 "
" " 8	1.2 "

The joints below the horizontal line are on the curve, while those above are on what is supposed to be perfectly straight track. It is possible in string lining to make all of the ordinates above the line equal to zero, thereby making a perfect tangent track and contributing materially to both the appearance and the riding qualities of the curve at the ends.

With the above in mind, let us assume we have an actual curve to measure. The steps are as follows:

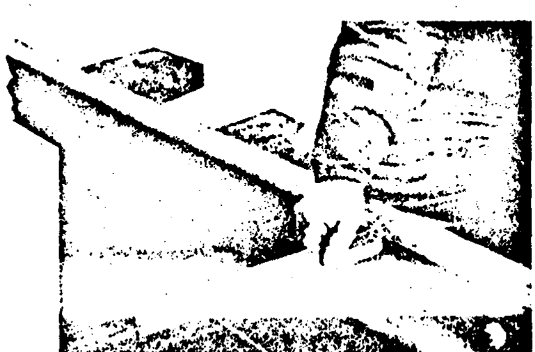
First, locate by eye the end of the curve; then go to that point, have one man hold one end of the string against the gage line of the high rail at the joint next farther from the curve than the joint to be measured. Another man holds the other end at the gage line of the high rail two joints nearer the curve. The third man then measures with a rule the middle ordinate at the intervening joint. If the ordinate is zero, and the track back of him (on the tangent) appears to be in fairly straight line, he can call this Joint No. 1. All three men then move up *one* joint nearer the curve, so that the ordinate at every joint is measured. (See Fig. 1.) Care should be exercised that no joints are omitted. A natural error for a beginner would be to move up a whole chord length, instead of half a chord length, as outlined above.

In order to identify the different joints, it is well to number them consecutively. It has proved very satisfactory to number the first joint and every fifth one thereafter until the end of the curve is reached. The last joint is given its proper number, whether that number is a multiple of five or not.

It is obvious from the foregoing description that three men will be needed to measure a curve. Only one of these men need be conversant with the principles of string lining, since the only duty of the other two is to hold the ends of the string tightly against the gage line of the high rail and to see that the string is properly taut at the time an ordinate is measured. Laborers from the section gang are gen-

Middle Ordinates of Different Chords for a One-degree Curve

Length of chord in feet	10	20	30	31	33	39	50	54	60	62	66	78	100
Middle ordinates in tenths of an inch	0.26	1.05	2.36	2.52	2.85	3.98	6.54	7.63	9.42	10.06	11.40	15.93	26.18



The Ruler Can Be of Any Desired Size and Shape erally selected for this duty because of their being close at hand and readily available for the short time required to measure the curve.

The man who measures the middle ordinate should select the points of beginning and ending of the curve, and should check the numbering of the rails. The numbers can be marked on the web or base of the rail with chalk or keel by one of the men holding the end of the string, preferably the man at the rear.

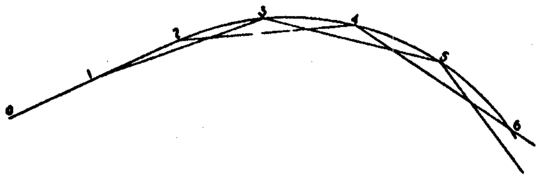


Fig. 1—Method of Measuring the Middle Ordinates of a Curve

The numbers indicate the string lining stations along the curve.
Joints or stations 0, 1 and 2 are on straight track.
The ordinate at station 1 is measured with one end of the string at station 0 and the other end at station 2.
The ordinate at station 2 is measured with one end of the string at station 1 and the other end at station 3.
The same procedure is used to take all the ordinates of the curve.

The recorder can then call off the numbers in some such fashion as: "Going to No. 45" or "You're at No. 45 now."

The recorder should provide himself with a sheet of paper upon which are ruled horizontal rows of lines. These he should number consecutively to correspond with the joint numbers. It is, of course, desirable for him to estimate beforehand the approximate number of rails in the curve, as this is a time-saver. Opposite each joint number is placed the middle ordinate, for the given chord, at that joint. This should be put down as soon as measured, to avoid the possibility of errors due to an effort to remember some of the ordinates.

Correctness of the original ordinates is a prime essential to the success of the string lining method, and care should be used to see that the measurements are uniformly accurate. Experience will show just what tension of string is required to secure an accurate measurement in the shortest possible time. If the string is not taut enough in the opinion of the recorder, he should call to one man to pull up on it slightly. If the ordinate changes, it is then apparent that the proper tension of the string had not been secured. Too great a tension will, naturally, break the string, but little fear is to be anticipated from this source after the proper pull is once ascertained. If the string does break, however, it should not be tied between the ends of the chord, as this obviously impairs the accuracy of the measured middle ordinates.

The Necessary Equipment

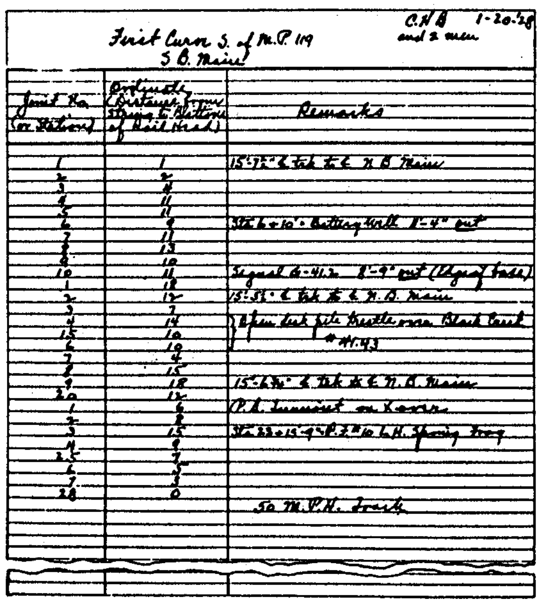
In order to obtain an unbroken length of string 60 ft. or more in length, it will usually be necessary to purchase 100 ft. of cord. The author's experience has been that all the ordinary varieties of cords purchasable at hardware stores or stationery stores will stretch appreciably at first. This need not be a source of worry, however, as long as the string is taut. A good plan to follow is to buy 100 ft. of string, such as that used by masons and bricklayers, tie both ends securely to wooden handles—which enable the ends to be gripped tightly and pressed securely against the gage of the rail—and then wind the unused portion on one stick. After a short period of use, the string will no longer stretch, and a fairly accurate chord will be determined by the length of the string itself, although this is not the guiding factor in fixing the proper chord length. Where all the rails are equal, the determining length is that of two rails.

Where the rails in the high rail of the curve are of different lengths, it will be necessary to chain or measure out the curve before taking any of the ordinates. This should be done with a steel tape or, if none is handy, with a metallic tape. Stations of 33 ft. are marked along the high rail, and numbered as before. The string should not be used for measuring the length of the curve because of the accumulative errors caused by stretching of the string and inequality of tension between different stations.

The ruler by which the ordinates are measured can be of any convenient size or shape. A triangular scale, of the type used by engineers or architects, makes an excellent rule, if the excess at one end is cut off so that the zero of the rule is at the end of the scale, although any rule whose graduations start at the end can be used. Whether an engineer's or an architect's rule is used depends upon the unit of measurement adopted. This leads naturally to the question of what is the best unit for the purpose.

Unit of Measurement of Middle Ordinate

Middle ordinates can be measured in any unit which appeals to the man who makes the calculations and takes the measurements of a curve. However, the selection lies almost entirely between the three



First Curve S. of M.P. 119
S.B. Main

C.H.B. 1-20-28
and 2 men

Joint No. (or Station)	Ordinate (Distance from String to Bottom of Rail head)	Remarks
1	1	15'-7½" ℄ Tak to ℄ N.B. Main
2	2	
3	4	
4	11	
5	11	
6	9	Sta 6+10' Battery Wall 1'-4" out
7	11	
8	13	
9	10	
10	11	Signal 6-41.2 8'-9" out (Edge of base)
1	12	
2	12	15'-5½" ℄ Tak to ℄ N.B. Main
3	7	
4	14	Open deck pile trestle over Black Creek
15	10	#41.43
6	10	
7	4	
8	15	
9	18	15'-6¾" ℄ Tak to ℄ N.B. Main
20	12	P.C. [illegible]
1	6	
2	8	Sta 23+15-9" P.S. #10 L.H. Spring Frog
3	15	
4	9	
25	7	
6	5	
7	3	
28	0	
		50 M.P.H. Track

Fig. 2—A Specimen Page of Notes

following units: (1) the tenth of an inch, (2) the eighth of an inch and (3) the hundredth of a foot.

Engineers are accustomed to measuring small quantities in hundredths of a foot, and this unit is suitable for practical purposes from their standpoint. However, the track foreman is not accustomed to this unit and, since he is vitally interested in the amount of throw required to correct the alinement of a curve it appears best to use either the tenth or the eighth of an inch. Further, the hundredth of a foot is not quite as accurate (by about 20 per cent, regardless of the relative numerical values of the two units) as either of the other two.

The eighth of an inch is a practical unit from the standpoint of the track man, and suitable enough for the engineer. The chief objection—and it is an important one—against its use is that middle ordinates must be written first as inches and eighths of an inch (as 4 7/8 in.) and then reduced to eighths only (as 4 7/8 in. becomes 39/8 in., or more simply, 39). The work of reduction assumes important dimensions as the curves become longer.

The tenth of an inch has the advantages of both the above units, and the disadvantages of neither. It is equally suited to the needs of the engineer and trackman, it is sufficiently accurate to permit of obtaining correct measurements and securing excellent results. It has, too, the big advantage that ordinates written in this unit can be written directly as whole numbers. For example, an ordinate of 5.4 in. can be written down immediately, without further reduction, as 54. The advantage of this feature can be appreciated only after the reader has actually lined one or two curves.

Practical Hints

1. When taking the data on the actual curve, the recorder should make pencil notes of the station numbers of fixed structures, located on or near the curve, which are apt to govern or restrict the permissible throwing of the track. For example, a trestle or steel bridge, the edge of a building, a deep cut, a high fill and other such obstacles form more or less serious impediments to the free throwing of a curve. Their position and extent should be noted by string lining station and plus.

The station and plus and the number of turnouts must also be noted for it occasionally happens that when a curve is re-alined, it it advisable or even necessary to change the number of one or more frogs and the corresponding turnouts.

2. The kind and quantity of ballast (per cent of standard ballast section) should be roughly noted down or remembered, in case it is necessary to throw portions of the curve a foot or more from their position before being re-alined. Although the road supervisor or road master is generally supposed to care foi such matters as ballast requirements, after he has seen the stakes or otherwise learned the amount of throw at the various points of the curve, it is well to be able to give an approximate estimate of the quantity of new ballast, if any, which will be required.

3. Mental note should be taken of the grade line of the track through the entire curve, especially at the entering and leaving ends of the curve, for reasons explained in a subsequent article.

4. Ascertain from a timetable or other reliable source the speed of trains operated over the curve to be lined, as their speed affects the spirals or easements to be used.

5. If certain ordinates measure a half unit more or less than a whole number (as 5.75 in.=57.5 tenths of an inch) attempt to balance them by increasing some by a half and decreasing others by a half, as the sum of the ordinates is a measure of the total angular change of alinement, and should be constant.

6. The recorder should call "all right" as soon as he has finished measuring an ordinate, so that all three men may move up to the next joint or station without loss of time.

7. It will sometimes happen that the greater part of a curve will have rails of uniform length, but that near one end will be a turnout or road crossing where some short rails will be found. In such a case, it is usually not necessary to chain or measure with a tape the remainder of the curve, if the measurements be started at the opposite end and carried toward the point where the short rails are located. For example, suppose a curve to have 67 rails, 57 of which are all 33 ft. long, after which there are 3 short rails, the remainder being again 33 ft. rails. Measure the rails from the end where the lengths are uniform, and then continue the stationing through the short rails by means of the string. This, of course, presupposes that the man pulling the string has learned very approximately the right amount of tension to give him a chord of exactly 66 ft.

8. On double or multiple track roads, it is wise to take track centers at intervals around the curve, in order that the man making the computations may know definitely whether a certain maximum throw will be such as to affect the clearance distance or minimum track centers.

9. On double or multiple track roads, it is always better to line separately the same curve on the different tracks than to line one track and attempt to line the others by maintaining a uniform center distance. There are several reasons for this. First, the existing track centers may not be uniform, as is usually the case, and a relatively small throw on the one track may entail a much larger (even a prohibitively large) throw on the next adjacent track. Second, most track foremen do not have steel tapes, but have cloth tapes which stretch as much as 3 or 4 in. in 25 ft.; consequently, the foreman cannot maintain really uniform track centers by means of such a tape. Third, it is nearly impossible to measure the center-to-center distance of curved tracks along the true radial line; and a small error in angular direction of measurement means an appreciable error in true radial distance. Fourth, the entering and leaving spirals are of different lengths and different spiral characteristics, which makes it impracticable to maintain uniform centers on spirals, even though the degrees of the circular curves are the same. For these reasons, where it is desired to line two or more tracks on the same curve, it is best to take the data separately for each curve as an entity.

10. Before going out to measure a given curve, the recorder or his superior should ascertain if any work is contemplated on the curve, and, if there is to be some work done, he should defer the measuring until such time as the track will remain undisturbed between the time of measuring it and the time of setting the stakes. It sometimes happens that data will be taken for a curve, and then a portion or all of that curve will be moved, relaid, or otherwise disturbed materially, rendering the data taken worthless—even a liability, in case stakes are set from throws computed from these data.

11. While the author strongly recommends using the tenth part of an inch as the unit for measuring the distance from the string to the gage line of the high rail, because of the fact mentioned above that such readings can be written as whole numbers without any further change (as, for example, 6.2 inches is 62), nevertheless, he is fully aware that most track men do not have a ruler graduated in

tenths of an inch and cannot easily secure one. Consequently, for their purpose it will be best to use an eighth of an inch as the unit, and then reduce the ordinates measured to whole numbers in order to work with them more easily. It is obvious that it requires some little thought to subtract $3\frac{7}{8}$ inches from $4\frac{1}{4}$ inches and get $\frac{3}{8}$ inch; and it is equally obvious that it is much easier to reduce $3\frac{7}{8}$ inches to 31/8 and $4\frac{1}{4}$ inches to 34/8 and then subtract 31 from 34, leaving $\frac{3}{8}$. The best way to do this is to make a little table, which can be referred to at any time, and which gives the "whole numerators" for the inches and eighths of an inch. The foreman should start out and make his own table, thus:

$\frac{1}{8}$ in.	1	1 in.	8
$\frac{1}{4}$ in.	2	$1\frac{1}{8}$ in.	9
$\frac{3}{8}$ in.	3	$1\frac{1}{4}$ in.	10
$\frac{1}{2}$ in.	4	$1\frac{3}{8}$ in.	11
$\frac{5}{8}$ in.	5	$1\frac{1}{2}$ in.	12
$\frac{3}{4}$ in.	6	etc.	etc.
$\frac{7}{8}$ in.	7		

Such a table will pay for the labor required to make it many times over. In fact, it is almost a necessity if the ordinates are measured in eighths of an inch.

12. Throughout this entire series of articles, wherever the word gage is used to indicate a point on the head or ball of the rail, it should be taken to mean the lowest point on the head of the rail on the gage side, rather than the theoretical gage line of the rail, which is about $\frac{5}{8}$ of an inch from the top of the rail. The reason for this is that almost all curved rails wear unevenly and the only point which is definitely determined is the lowest point of the head, where the wheels do not touch or at least touch but a little.

13. The foreman must not get the idea that this system will work only for 33-foot rails. Any chord length at all will do, just so the same chord is used to measure the entire curve. If a curve happens to be laid with sawed rail only 27 ft. long, then the proper chord to use is twice 27 or 54 ft. As far as the foreman or trackman is concerned, one chord length is just as good as another; but for the purposes of any engineers using the method, the best length is 33 ft., because all the tables giving the engineering data have been worked out for that length rail (that is, for a 66-ft. chord), and considerable time and computation will be saved when figuring central angles, spiral functions, etc. For relining purposes only, however, no tables are needed and any length will do.

The Arithmetic of String Lining Curves

High Grade Passenger Service Demands Good Curve Alinement

IN THE TWO previous articles of this series the manner of taking the measurements of an actual curve has been presented. In this article it is assumed that it has been decided to change the actual curve in certain ways so as to rectify any errors of curvature that may exist, such as sharp and flat spots. This article will set forth the fundamental principles underlying the means of effecting the change from the actual curve to the revised one. In the following discussion, no mathematical analysis is given for any of the principles stated, except the simple geometrical proof for the theorem regarding the throwing of a joint or station, although complete and satisfactory proof can be given mathematically for all of the rules given. These proofs are omitted from this series of articles as it is thought that they will not be desired by most of the readers.

Sum of Ordinates Must Remain Constant

The first principle of string-lining is that the sum of the ordinates of a curve must remain constant throughout any series of operations designed to correct the alinement of that curve. In other words, if a curve is measured and the ordinates taken in the field total 656 tenths (or eighths, quarters or any other units) of an inch, the total 656 must remain the same for any revised curve. It can be proved that this total represents the actual angle between the two tangents or pieces of straight track at each end of the curve; and inasmuch as it is obvious that the angle between the straight tracks cannot be changed, the total of the measured ordinates must not be changed.

The second principle is that the sum of the errors between the figures for the original curve and the figures for the revised curve must equal zero, the error at any station being defined as the difference between the original ordinate and any revised ordinate selected by the man lining the curve. It is, briefly, the original ordinate less the revised. Suppose that at some station of a curve the ordinate measured in the field is 48 eighths of an inch, or 6 in. If, for reasons explained later, it is desired to change this ordinate when re-lining the curve, to some such figure as 41 eighths of an inch (or 5⅛ in.) the *error* at that station will be 48 less 41 or 7 eighths of an inch. If the revised ordinate is larger than the original, the difference or error is termed negative and a minus sign placed in front of it. For example, if the revised ordinate in the above case happened to be 53 instead of 41, the error would be 48 less 53 or minus 5 eighths of an inch. In view of the above explanation, it will be clear to the reader that, if the total of the revised ordinates must equal the total of the original ordinates, the total difference between the two must be zero, the total difference being merely the sum of the separate differences or errors at each station. In other words, the second principle is practically the same as the first, but is merely a more convenient way of expressing it.

Effect of Throw on Adjacent Stations

The third principle is very important, and should be thoroughly understood before the reader attempts to line any curve. It is as follows:

Rule: If one joint (or station) of a curve is moved in or out a certain distance (which distance is called the throw), the middle ordinates at the stations on each side of the one moved will be changed by half the amount of the throw, and in the direction opposite to the throw.

The above rule is best explained by a diagram. Let

*This is the third of a series of six articles on the string lining of curves, describing the manner in which the line can be corrected by the track men without the use of instruments other than a piece of string and an ordinary rule. The first article of this series, which appeared in the January issue, page 4, presented the merits of this practice in contrast with the use of a transit. The second article, which was published in the February issue, page 62, describes the methods of taking the measurements. The fourth and fifth articles, which will appear in the May and June issues, respectively, will describe the method of selecting the ordinates for the revised curve and the sixth and concluding article will describe the manner of placing the stakes preliminary to the lining of the curve.

THE STRING LINING OF CURVES

the reader refer to the accompanying figure, in which is shown a portion of a circular curve ABC, whose chord is AC and middle ordinate MB. The arc ABC can be taken, if desired, as the length of two rails of the high side of a curve, with AB as one rail and BC as the other, with the joint at B. It is plain, therefore, that as a practical matter, the rail BC can be moved without disturbing the rail AB. Therefore, consider that the joint (or station) C is moved or thrown out—that is, away from the center of the curve—to the point D, so that the rail BC now occupies the position BD. The new chord (or line representing the position of the string) is now the line AD, and the new middle ordinate is BE. In actual practice, the length BE is so closely equal to MB (the original middle ordinate) less MF that the difference is negligible. But MF, from geometry, is almost exactly one-half of CD, the amount of throw. This is because in the triangle ACD, M is the middle point of AC, and therefore AM is one-half of AC, so that MF is one-half of CD. Therefore, BE equals MB less MF; or, in words, the ordinate *after* throwing equals the ordinate *before* throwing, less one-half of the throw. In the same way, it can be shown that the joint (or station) the other side of C (on the right) is similarly affected. Hence, an out throw at any joint increases the ordinate at the joint by the full amount (CD in the diagram) and decreases the ordinates at each of the joints on the two adjoining sides by one-half the amount of the throw. Conversely, if the throw is in, the ordinate at the station thrown is decreased by the full amount of the throw, and the ordinates at the stations on each side are increased by one-half the amount of the throw.

From the above demonstration it is seen that the operation so described consists in the *addition* or *subtraction* from the measured middle ordinate at a station of a certain quantity, known as the throw, and in the *subtraction* or *addition* of half that amount to the ordinates at the two stations on either side of the one thrown.

A Simple Example

For example, suppose that at any three consecutive stations or joints of a curve we have the following three ordinates:

18	24	18

These indicate a typical sharp spot, which, of course, it is desired to eliminate by making all three as nearly equal as possible, and thus making a uniform curvature. We can "throw" the middle joint in by subtracting a certain amount and adding half as much to the two figures 18 on each side. Note: It is obvious that the 24 must be reduced and the two 18s increased, so that all three can be made equal.

We can subtract 4 from 24, which will add 2 to both figures 18. We then have:

18	24	18
+2	—4	+2
20	20	20

Thus, we have equalized all three ordinates and made this section of the curve a true circle, which is one of the purposes of string-lining.

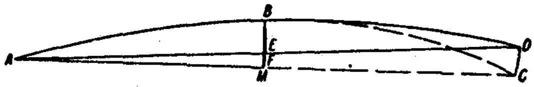
The process of lining a curve (on a slate or a sheet of paper) consists in repeating the above operation at every station, with certain modifications as explained later. It is logical to conclude that from the simplicity of the above operation there must be some definite relation between error and correction; and, in fact, such is the case. However, it is not the purpose of this article to give a detailed mathematical analysis, but simply a set of working rules for the performance of the mechanical operations of string-lining.

In connection with the above rule, it should be noted that the throws at the first and last stations of a curve must be zero. If the first or last stations are thrown, the resulting half-throws will be imparted to the tangent or straight track, throwing it out of alinement. For example, if the first station of a curve has an ordinate of 0, and we throw the track 2 (units of any kind) out, the straight track at the left of the 0 will have a half-throw of -1, which will put a small kink in it. Consequently, we must correct the first station by throwing the second, making the throw such that the half-throw at the first station will correct it.

Relation Between Error and Throw

If, at each station, we add all the errors to and including that station and write them down opposite their station numbers, we obtain a column which we can call the "Sum of Errors." If now we obtain the sum of all of these from the first station to any given station, and write that sum opposite the *following* station, we obtain the half-throw required at that following station.

Let us suppose a curve having 53 stations, opposite each of which is written an actual and a revised ordinate, in two columns, and in the third column the differences, or errors, between the actual and the revised ordinate at each station. Let us now put down opposite the first station the error at that station; opposite the second station, the sum of the first and the second errors; opposite the third station, the sum of the first three errors; and so on, until at station 53, we have the sum of the errors for the entire curve, which must, of course, equal zero. The addition of the errors is algebraic; that is, due regard is taken of the sign of the error. For example, if the first error is (5), the second (-3), and third (-6), the sums of the errors at the first three stations are, in order (5); (5)+(-3)=(2); and (5)+(-3)+(-6)=(-4). It is, of course, not necessary to add all the errors every time, since the sum at any station is obtained directly by adding to the sum of the errors at the previous station, the error at the station. For example, the above would be obtained actually as 5; 5-3=(2); 2-6=(-4); etc., etc.



The Effect of Throw on Adjacent Stations

Having obtained the column headed sum of errors, we next add this column to and including each station and bring the total into the next column under the *following* station. To illustrate (remembering that the throw at the first station is zero), we have for the above figures, the following half throws:

First station		0
Second station	0+5	= +5
Third station	0+5+2	= +7
Fourth station	0+5+2—4	= +3

Once again, it is not necessary to add the figures from the first station on, since the same result is obtained

THE STRING LINING OF CURVES

more easily by adding to the sum previously obtained the adjacent figure in the next row. For example:

First station		0
Second station	0+5	= +5
Third station	5+2	= +7
Fourth station	7—4	= +3

A resumé of the above operations may make the process clearer. Consider the following hypothetical curve:

Station or Joint Number	Actual Original Ordinate in Tenths of an Inch	Revised Ordinate for Re-alined Curve	Error	Sums Errors	Half Throw
1	1	0	+ 1	+ 1	0
2	3	2	+ 1	+ 2	+ 1
3	7	4	+ 3	+ 5	+ 3
4	3	6	- 3	+ 2	+ 8
5	4	8	- 4	- 2	+10
6	7	8	- 1	- 3	+ 8
7	10	8	+ 2	- 1	+ 5
					+ 4

In the first column are the station numbers.

In the second column are the middle ordinates of the actual curve, measured in the field as outlined in the second article.

In the third column are the revised ordinates selected for the curve, in accordance with certain principles explained later.

In the fourth column are the errors, e, obtained as explained above. For instance, at station 3, 7—4=3; at station 5, 4—8=-4; etc.

In the fifth column are the sums of the errors to and including each station. These are obtained by writing down the error at the first station, and then at each station thereafter writing down the sum of the preceding figure and the figure on the next line in the column headed "Error." Thus writing down 1 in the first station, we next add 1 and the 1 shown on line 2 as error at station 2. This makes 2. To this 2, we add the 3 on the next line below, in the error column, making a total of 5.

In the sixth column are the half-throws. To obtain these we add horizontally instead of diagonally downward as we previously did to obtain the sum of the errors. Thus, 0+1=1, at station 2; 1 plus the 2 at station 2, is carried as 3 and written under station 3; 3 plus the 5, at station 3, is written as 8 at station 4, etc. The arrows indicate the direction of the additions and the position of the results.

In this way a final half-throw is obtained at the last station. This half-throw must be zero, as we have seen above. However, because it is nearly impossible to pick the correct ordinates at the first trial, this half-throw will not generally be zero, and we must make it so.

It can be shown that the sums of the products of the error at each station by that station number must be equal to zero, which is a condition that the curve is lined. That is, if at a given station the error (difference between the actual and the revised ordinate) is -5, and the number of the station is 21, the product of these two quantities, is -105. The sums of all such products for the entire curve must equal zero as a condition that the curve is really lined.

It can likewise be shown that the sum of all these products is equal to the half-throw at the last station, whether that half-throw is zero or not. This being the case, it is immediately evident that if the final half-throw is not equal to zero, it can be made so by making equal to zero the sums of the products above referred to.

If we have a certain curve, of any number of stations, and we find that the final half-throw is not equal to zero, but some such number as, say, 46, let us consider a way of changing that 46 to zero. Let us say that the ordinate at the 5th station is 23 on the original curve and 25 on the revised curve. The difference is -2; the product of -2 and the station number, 5, is -10. Now let us say that the ordinate at some other station, as the 37th for instance, is 31 on the original curve and 26 on the revised curve. The error is +5; +5 times 37=+185.

If, now, we change the revised ordinate at station 5 from 25 to 24, and that at station 37 from 26 to 27, we have kept the total of the ordinates the same, for we have subtracted one at station 5 and added one at station 37. But what effect has this had on the products of the error and station number? The error at station 5 is decreased from -2 to -1, making the product now -5 instead of the previous -10; at station 37, the error is decreased from +5 to +4, making the product now 148, instead of the previous 185. Thus, while maintaining the total of the ordinates of the revised curve the same, we have changed the total of the products from the first +175 (-10+185) to the second, 143 (148—5); that is, we have *decreased* the final half-throw by an amount equal to 175—143, or 32. This leaves the final half-throw now 14 (46 -32). The same process can now be repeated, the only difference this time being that the station numbers of the ordinates changed must be different. The reader will note that the net difference in the sums of the products, which amounts to 32, is the difference between the station numbers of the revised ordinates which were changed; that is 32=37—5. This will always be the case. Consequently, to diminish the final half-throw by the remaining 14, we have but to choose two stations whose numbers differ by 14, and repeat the above process, with the assurance that the final half-throw will now be zero, and the curve will be lined.

The rule for changing the final half-throw to zero can now be announced. It is:

When the final half-throw is positive, subtract from the revised ordinates having high station numbers and add an equal amount to the ordinates having low station numbers, choosing stations in pairs such that the sum of the differences of the station numbers taken in pairs equals the numerical amount of the final half-throw. When the final half-throw is negative, reverse the procedures, subtracting from the ordinates having low station numbers, and adding to those having high station numbers.

Thus, if we have a curve of 63 stations, wherein the final half-throw, obtained with a trial set of revised ordinates, is 59, we could add 1 to the revised ordinate at station 6 and subtract 1 from the revised ordinate at station 48, thereby making a difference of 48-6 or 42, in the final half-throw. We could then repeat the process, choosing this time stations 8 and 25, whose difference is 17, which difference, added to the previously obtained 42, makes the total of 59 and reduces the final half-throw to 0.

The reader will note that the only condition attached to the above method of change is that the sum of the differences of the station numbers of the revised ordinates changed must equal the final half-throw. The operator, therefore, is wholly at liberty to choose the stations which he will change. This is a valuable property of the system—one which cannot be over-emphasized, as its understanding enables the operator to obtain a much better solution than he otherwise would be able to achieve.

Determining the Throw for a Curve*†

The Explanation of the Steps Entering Into the Selection of the Ordinates for the Revision of Line

IT IS a well known fact that a spiral or some other sort of easement curve is necessary to connect a circular curve with a tangent at each end. There are several reasons for this, chief of which is the fact that if the spiral was not there, there would be a considerable jolt as the train passed from the straight track to the curve.

Various forms of easement curves have been established and have enjoyed more or less extended usage. The two commonest forms of easement, however, are the Talbot spiral, and the cubic parabola. The mathematical properties of these two curves, insofar as they concern string lining, are so nearly the same that they may be considered identical for the purposes and scope of this discussion. As is the case with the circular curve, there is an analogy between the transit deflections, and the middle ordinates measured with a string. However, it is obvious that the ordinates at the several points of the spiral will not all be equal, but will vary according to some rule or law. It can be demonstrated that the ordinates vary according to the law:

1/6, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc., etc., where the ordinate at the point of spiral is 1/6.

Since for all practical purposes the first ordinates and the second also are usually quite small the first ordinate in the above series can be written as 0 without material error. Thus, in order to have a true spiral it is only necessary to vary the stringline ordinates in a series whose terms are multiples of the natural numbers in their proper order.

*This is the fourth of a series of six articles on the string lining of curves, describing the manner in which the line can be corrected by the track men without the use of instruments other than a piece of string and an ordinary rule. The first article of this series, which appeared in the January issue, Page 4, presented the merits of this practice in contrast with the use of a transit. The second article, which was published in the February issue, Page 62, describes the methods of taking the measurements. The third article, which appeared in the April issue, Page 168, presented the basic principles underlying this method of determining curve alinement. The fifth article, which will appear in the June issue, is a continuation of the article in this issue discussing the method of selecting the revised ordinates for a curve to give the minimum throw. The sixth and concluding article will describe the manner of placing the stakes preliminary to the actual lining of the curve.

In previous articles we have considered the methods of making the computations required to line a given curve; that is, we have established rules for the mechanical operations which are absolutely the same for all curves, no matter what the degree of curve or how bad the alinement. The next question arising is therefore: How shall the revised ordinates be selected so that the best solution for a given curve is obtained? We have seen that, since the choice of the ordinates to be used for the revised curve is left absolutely to the discretion of the operator, there are several different solutions for any given curve. The problem is to pick the *best* one. This will depend largely upon what is meant by the term "best."

Depends on What Is Wanted

If that solution is desired which will make the maximum necessary throw a required amount (which is by far the commonest case in actual practice), one line of work will be followed. If it is desired to have a perfectly regular curve, or to eliminate certain branches of a compound curve in order to make a simple curve, it will generally be necessary to sacrifice small throws and use large throws to accomplish the result; accordingly, a different method of attack must be followed. If it is required to install a spiral in an existing unspiraled curve, the line of procedure is again changed. As still another example, if it is required to install a spiral which can be operated by trains at a certain fixed maximum speed (given by the railway timetable), the solution will be obtained in another manner.

It might be thought at first glance that the logical method to use in beginning to line a curve on paper would be to take the original ordinates as measured in the field, pick out a spiral to fit each end of the curve, subtract the sums of the ordinates used in these two spirals from the total of the original ordinates and divide the remainder by the number of ordinates remaining in the curve. For example, if

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we assume a curve of 50 stations, the total of original ordinates for which is 978, we might pick a spiral for the beginning end (say, for purposes of this illustration, 0, 3, 6, 9, 12, 15, 18, 21) and a spiral for the leaving end (say, a spiral such as 0, 4, 8, 12, 16, 20). The sum of the ordinates in the first spiral is $3+6+9+12+15+18+21$ which equals 84; in the same way, the sum of the ordinates of the second spiral is 60. Thus, the sum of the ordinates for the two spirals is $84+60$ which equals 144. From our first principles, we have the theorem that the sum of the ordinates of the revised curve must equal the sum of the ordinates of the original curve. Hence, we have as the sum of all the ordinates of the revised *curve proper* (that portion between the two spirals), a total of $978-144=834$. But the first spiral occupies 8 stations; the second occupies 6; so that the two together occupy 14 stations. This leaves $50-14=36$ stations remaining for the circular curve itself. Thus, we see that the total of the 36 stations must be 834, which gives us an average ordinate of $834/36=23\,6/36$. We could, then (other things being disregarded), obtain a correct total for the revised ordinates by using 30 ordinates of 23 and 6 ordinates of $24=690+144=834$.

May Require Excessive Throwing

The foregoing procedure, while (as pointed out above), appearing to a beginner as being more or less the logical thing to do, does not as a rule give a curve which can be obtained from the existing curve without excessive throwing of the original curve. The reasons for this can scarcely be understood unless the theory of string lining is studied.

Accordingly, we must seek another method. Actual practice in lining curves on paper has revealed that the best method consists in trying a certain spiral and a certain average ordinate for the circular curve, and in watching the throws as the lining progresses. That is, given a set of figures representing a curve, the operator will select a spiral, write it down and figure the throws it entails as he puts it down. Then he will progress to the circular curve, picking a trial ordinate and computing the required throws as he goes along. Of course, the selection of the proper spiral will be rather a hard task for a beginner, but it will be found that for each curve there is only a very limited number of spirals which can be used without entailing prohibitive throws. After having lined one or two curves, the operator will be able to see practically at a glance the spiral which best fits the curve.

Procedure to Be Followed

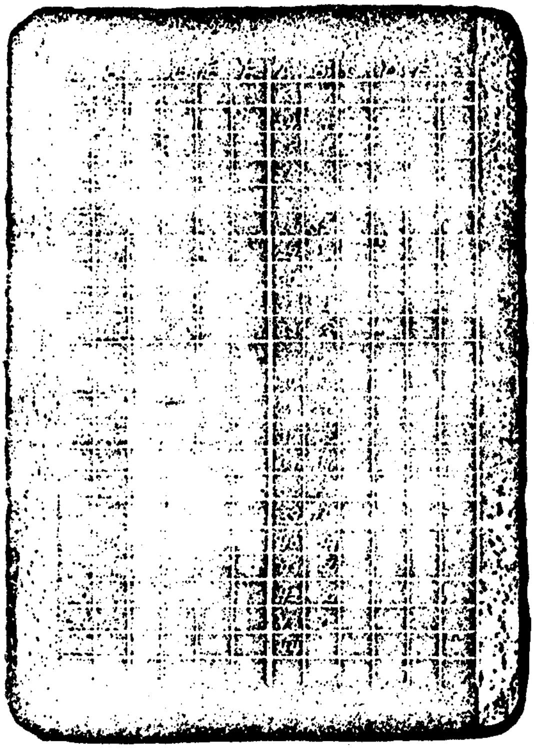
The reader should, by this time, be prepared to follow the steps in the solution of an actual numerical curve, inasmuch as he should now be familiar with the mechanical processes to be followed, once the revised ordinates have been selected. Since the commonest case met is that wherein the operator attempts to keep the throws below a certain arbitrary limit, we shall first take up this case.

In the curve in question, there are 43 stations. The chord length used was 66 ft. (length of two 33-ft. rails), and the ordinates were measured in tenths of an inch, and recorded as whole numbers, each unit number shown representing one-tenth of an inch. Thus, the ordinate 15 at station 10 represents an ordinate of an inch and a half.

Let us assume that the limiting throw has been fixed at 6 in. in both directions; that is, the actual curve may be thrown either 6 in. in (toward the center of the curve) or 6 in. out (away from the center of the curve). Since the figures we obtain are always exactly one-half the actual required throws, we shall have the condition that our half-throws cannot exceed + or — 3 in. Written as tenths of an inch, this means that our largest throws cannot be greater than + or — 30.

The Effect of Changing an Ordinate

An explanation of the actual effect of changing an ordinate should be explained here. Suppose we have a curve wherein the ordinate which seems to predominate in our solution of the revised curve is 42. If it appears, as we work along, that the throws are going to run up too high in a positive direction,



A Slate Will Be Found Convenient

[Note: One applying this method of lining curves will find it a great saving of time and labor to write down the original ordinates on a slate which has been ruled off into lines and columns. The work of making the changes necessary to the calculation of a curve is of such a character that erasures must necessarily be easy to make. The author has found that a slate, ruled into six vertical columns (one column each for A, B, C, D, E, and F) and with horizontal lines at intervals of about a quarter of an inch, makes an admirable board upon which to make the necessary changes and calculations. The lines should be cut into the slate with a knife or other sharp instrument, and station numbers should be scratched in with a pin.]

which means that the revised ordinate is too small, we can easily change the ordinate to 43. Either one or several ordinates can thus be changed. Frequently it is necessary to change only one or two of the ordinates to obtain the proper reduction in throw. This procedure, in effect, is equivalent to compounding the curve very lightly for a short distance, such as two or three rail lengths. For a 66-ft. chord,

THE STRING LINING OF CURVES

changing the middle ordinate one-tenth of an inch changes the degree of curvature 5 min. and 15 sec. In the case above, if we changed three 42s to 43s, we would be compounding the curve from 3 deg., 40 min. and 30 sec. to 3 deg., 45 min. and 45 sec., for a length of 99 ft. Actually, on the track, no change of curvature would be apparent. Here, then, is one of the most valuable features of string-lining. By lightly compounding a curve for a short distance, we can cut down the throws required to line the curve, and yet not affect the riding qualities adversely. This could not be done with a transit except at great expense of time and labor.

With these points in mind, let us glance at the following curve.

On the slate the following notation has been used for brevity:

A is the station number of any station.
B is the original ordinate at each station.
C is the revised ordinate at each station.
D is the error or difference, B minus C, at each station.
E is the sum of the errors, to and including each station.
F is the half-throw at each station.

Station Number (A)	Original Ordinate (B)	Station Number (A)	Original Ordinate (B)
1	0	23	17
2	5	24	34
3	7	25	18
4	9	26	13
5	9	27	10
6	16	28	32
7	34	29	18
8	23	30	20
9	12	31	20
10	15	32	33
11	25	33	13
12	20	34	14
13	13	35	31
14	19	36	22
15	28	37	17
16	25	38	17
17	28	39	16
18	24	40	4
19	16	41	8
20	8	42	2
21	13	43	0
22	17		

Selection of Initial Spiral

Let us, first of all, consider the spiral which best fits the entering end of the curve. The first step is to make a mental note of the point where the spiral appears to end, and of what the average ordinate for the entire curve seems to be. It is evident immediately that the spiral must start at Station 1, and must end either at Station 6 or Station 7. Inspection of the ordinates of the original curve shows that the average ordinate for the revised curve will have to be approximately 20, or close to 20, either one way or the other.

If we try a spiral with a factor of 4, we shall have as the revised ordinates, 0, 4, 8, 12, 16, 20. Let us try this and see what the consequent throws will be at the point where the spiral ends and the circular curve begins.

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
1	0	0	0	0	0
2	5	4	1	1	0
3	7	8	— 1	0	1
4	9	12	— 3	— 3	1
5	9	16	— 7	—10	— 2
6	16	20	— 4	—14	—12
					—26

The throw at the first station must be zero, on all curves, for the reason explained above. Accordingly, we write 0 at the first station, and proceed to compute the throw at the second station. At Station 2, the original ordinate is 5 and the revised ordinate is 4, which makes the difference, D, equal to +1. Since the error at the first station was 0, the sum of the errors to and including Station 1 is 0, and to and including Station 2 is 0+1=1. The throw at Station 2 is obtained by adding to the throw at Station 1 the sum of the errors in the opposite column. That is, 0+0=0, and this second zero is brought down in the throw column as the throw at Station 2. In this same way, the throw at Station 3 is obtained by adding to the 0 in the throw column at Station 2 the 1 in the sum of errors column at Station 2 and bringing the sum down in the throw column at Station 3.

Continuing in this fashion we arrive at a half-throw of —26 at Station 6. This means that the full throw at Station 6 is —5.2 in., which is nearly the limiting throw fixed at the outset. Consequently, we look at the next station to see what is going to happen there, and to see if the throw is going to run over our limit. The revised ordinate of 20 at Station 7 will give us an error at Station 7 of 34—20, or +14. This +14, added to the —14 which was the sum of the errors to and including Station 6, reduces the sum of errors to 0, thus making the throw at Station 8 the same as at Station 7. Accordingly, we shall try the revised ordinate of 20 for a few stations and see if it is suitable.

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
7	34	20	14	0	—26
8	23	20	3	3	—26
9	12	20	— 8	— 5	—23
10	15	20	— 5	—10	—28
					—38

It is evident, from the —38 at Station 11, that we have exceeded our limiting half-throw of 30. Now, we can change the half-throws and bring them within the assigned limits in several ways. For example, we have the choice either of changing the spiral in such a way as to reduce the negative throws, or we can change the ordinates of 20, making some or all of them, since so far there are only a few, 19. Let us try this latter method first. We have

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
7	34	19	15	1	—26
8	23	19	4	5	—25
9	12	19	— 7	— 2	—20
10	15	19	— 4	— 6	—22
					—28

We have, then, by changing the revised ordinate from 20 to 19, reduced the negative throws until they are less than the assigned limit. However, the last throw, at Station 11, is so close to —30 that we should examine the next one or two stations to see whether there is a possibility that the throws will exceed —30. We see that at the next station the original ordinate is 25, which gives as the error +6, thus making the sum of the errors at Station 11 equal to zero. Consequently, the throw at Station 12 will be —28+0=—28. Now examine the following station. We see that the error will be +1, thus making the sum of errors equal 1; the throw at Station 13 will then be —27. However, at Station 13, the original ordinate is only 13, making the error —6, which will make the sum of errors at Station 13=-5, and the throw at Station 14=-32. The work is as follows:

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
11	25	19	6	0	—28
12	20	19	1	1	—28
13	13	19	— 6	— 5	—27
14	19	19	0	— 5	—32.
					—37

Here, again, we have over run. Let us, therefore, try the method of reducing the spiral. Try a 0, 3, 6, 9, 12, 15, 18, (and, if necessary, 20 as an approximate final term) spiral. The work is as follows:

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
1	0	0	0	0	0
2	5	3	2	2	0
3	7	6	1	3	2
4	9	9	0	3	5
5	9	12	— 3	0	8
6	16	15	1	1	8
7	34	18	16	17	9
					26

It is immediately evident as soon as this point is reached that, with such a spiral, the throws will far exceed the assigned limit in the positive direction. Accordingly, we have now the information that a spiral of factor 4 is too large and a spiral of factor 3 is too small. It is obvious, therefore, that some spiral in between these two figures must be used to obtain good results. Any fractional multiplier can be used to obtain a spiral; but, since fractional ordinates are not used, we must add or drop the fractional part of the units in writing down the spiral. For example, a spiral of factor 2/3 would, theoretically, be as follows:

0, 2/3, 1 1/3, 2, 2 2/3, 3 1/3, 4, etc.

But, in order to have whole numbers, we may write this spiral in a number of different ways, choosing for our use that one which best fits the curve upon which we may be working at the time. Thus, we may write

0, 1, 1, 2, 3, 3, 4, etc.

If we choose a spiral of factor such as 4½ we may write either of the following:

(a) 0, 4, 9, 13, 18, 22, 27, 31, 36, etc.
(b) 0, 5, 9, 14, 18, 23, 27, 32, 36, etc.

In (a), the halves are dropped, and only the lower numbers used, while in (b) the halves are added to those figures in which they occur. Either of these may be called, with equal accuracy, a spiral of factor 4½—or, more simply, a 4½ spiral.

In the curve with which we are dealing, it was evident, from the above calculations, that the 4 spiral was better suited to the curve than the 3 spiral, since the sum of the errors did not increase as rapidly. Let us, therefore, try a 3½ spiral, adding the halves, as follows:

0, 4, 7, 11, 14, 18, etc.

We have the following:

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
1	0	0	0	0	0
2	5	4	1	1	0
3	7	7	0	1	1
4	9	11	— 2	— 1	2
5	9	14	— 5	— 6	1
6	16	18	— 2	— 8	— 5
7	34	20	14	6	—13
8	23	20	3	9	— 7
9	12	20	— 8	1	2
10	15	20	— 5	— 4	3
11	25	20	5	1	— 1
12	20	20	0	1	0
13	13	20	— 7	— 6	1
14	19	20	— 1	— 7	— 5
					—12

So far, the spiral and average ordinate picked out seem to fit the curve quite well. We shall go on with the same average ordinate, computing the throws as we proceed.

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
15	28	20	8	1	—12
16	25	20	5	6	—11
17	28	20	8	14	— 5
18	24	20	4	18	+ 9
19	16	20	— 4	14	27
					41

The Throw Is Excessive

At this point, we exceed the limiting throw in the positive direction. Since the first part of the curve fits so well, let us attempt to leave this part of the curve alone, and try to change the revised ordinates in such manner as to bring the 41 below 30. Let us change a few of the ordinates of 20 to 21, starting at some such place as Station 14. Try two ordinates of 21, at Stations 14 and 15, after which again change back to 20. We have

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
14	19	21	— 2	— 8	— 5
15	28	21	7	— 1	—13
16	25	20	5	4	—14
17	28	20	8	12	—10
18	24	20	4	16	+ 2
19	16	20	— 4	12	18
20	8	20	—12	0	30
21	13	20	— 7	— 7	30
22	17	20	— 3	—10	23
23	17	20	— 3	—13	13
24	34	20	+14	1	0
25	18	20	— 2	— 1	1
26	13	20	— 7	— 8	0
27	10	20	—10	—18	— 8
28	32	20	12	— 6	—26
					—32

Here, again, we over run. We shall try changing some of the 20s back to 19s, starting with Station 24 (selected at random, as were the others). It is evident, by an inspection of the rate at which the column headed sum of errors is changing that we shall need three or four 19s to reduce the negative throw properly. For we are carrying a negative total of —6 in the sum of errors column. If the reader will observe the following original ordinates, he will see at once that there is an 18, a 20, another 20, before the ordinate 33 is reached at Station 32. Thus, if we have a negative throw when entering this group, and our sum of errors is negative, we shall keep on adding to the negative throw, with an almost undiminished total error, inasmuch as the three ordinates mentioned are all close to the average of the revised curve and will not materially affect the total of the sum of errors.

Inserting four 19s, Station 24 to 27, inclusive, we have the following:

THE STRING LINING OF CURVES

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
23	17	20	—3	—13	13
24	34	19	15	2	0
25	18	19	—1	1	2
26	13	19	—6	—5	3
27	10	19	—9	—14	—2
28	32	20	12	—2	—16
29	18	20	—2	—4	—18
30	20	20	0	—4	—22
31	20	20	0	—4	—26
32	33	20	13	9	—30
33	13	20	—7	2	—21
34	14	20	—6	—4	—19
35	31	20	11	7	—23
36	22	20	2	9	—16
37	17	20	—3	6	—7
38	17	20	—3	3	—1
39	16				2
40	4				
41	8				
42	2				
43	0				

The End of the Curve

At this point, it is time for us to consider the proper spiral for the leaving end of the curve. We have reached the point where the circular curve abviously ends, and the spiral begins. We are carrying at this point only a very small throw and also a very small sum of errors, so that the total of the ordinates for our spiral will need to be almost the same as for the actual curve. Strictly speaking, it is exactly the total of the remaining ordinates of the original curve plus the sum of the errors at our last station; that is, plus 3. Let us try a spiral such as 0, 5, 11, 16, 21. This will necessitate the changing of the last shown 20 to a 21, but that will not require much additional labor. Strictly speaking, the above spiral should read, 0, 5, 11, 16, 22 (such that the differences between the ordinates, taken two at a time, will read, 5, 6, 5, 6); but such a true spiral would necessitate the installation of an ordinate of 22, which appears nowhere alse in the curve, although not objectionable from the standpoint of actual track layout. This spiral, then gives us

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
38	17	21	—4	2	—1
39	16	16	0	2	+1
40	4	11	—7	—5	3
41	8	5	3	—2	—2
42	2	0	2	0	—4
43	0	0	0	0	—4

We have reached the end, and the total of the sum of errors column is zero, which proves that the total of the revised ordinates is the same as the total of the original ordinates. We have, however, a residual half-throw of —4, instead of the zero which is required as a condition that the curve is fully lined. It is necessary for us to change this —4 to 0. Remembering the rule given in the previous article, that when the residual half-throw at the end of the curve is *negative,* we subtract from the ordinates near the top of the curve (those having low station numbers) and add to those nearer the bottom of the curve, in such manner that the difference between the station numbers of the ordinates changed, when multiplied by the number of units change, equals the final residual half-throw. Applying this rule to the —4, we see that we must subtract one from a station somewhere, and add one to a station four stations nearer the bottom of the page. We are entirely at liberty to pick any two ordinates four stations apart; but if the reader will examine the curve, he will see that if we add to Station 27 and subtract from Station 23, we shall not introduce any more 21s or any 18s into the curve, as we would do if we picked up almost any other stations. Consequently, we shall subtract one from the ordinate at Station 23, and add one to that at Station 27. We then have the following:

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
23	17	19	—2	—12	13
24	34	19	15	3	1
25	18	19	—1	2	4
26	13	19	—6	—4	6
27	10	20	—10	—14	2
28	32	20	12	—2	—12
29	18	20	—2	—4	—14
30	20	20	0	—4	—18
31	20	20	0	—4	—22
32	33	20	13	9	—26
33	13	20	—7	2	—17
34	14	20	—6	—4	—15
35	31	20	11	7	—19
36	22	20	2	9	—12
37	17	20	—3	6	—3
38	17	21	—4	2	3
39	16	16	0	2	5
40	4	11	—7	—5	7
41	8	5	3	—2	2
42	2	0	2	0	0
43	0	0	0	0	0

The final residual half-throw is now zero, and the entire curve is lined. The predominating ordinate of the revised curve is 20, with but very few variations therefrom. But the reader will doubtless see, by this time, just what a saving in throw was effected by the insertion of the two ordinates 21 instead of 20s at Stations 14 and 15, and by the four ordinates of 19 at Stations 23 to 26, inclusive. Such a slight compounding could not possibly have been obtained by the use of a transit, for, with the instrument, it would have been necessary to install a curve of 1 deg. and 45 min. for a portion of the curve, and then to have compounded the remainder to something like 1 deg. and 30 min.

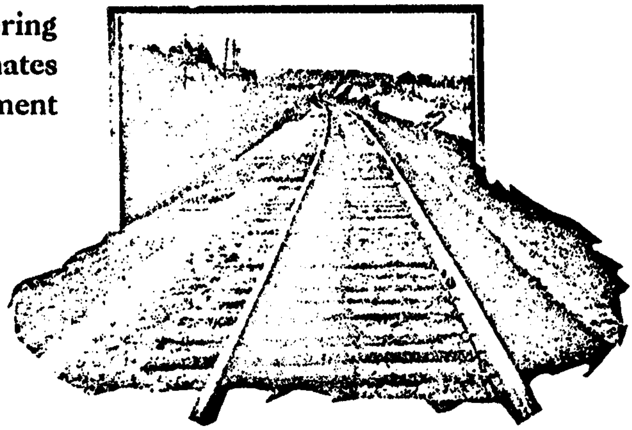
Requires Less Throwing

In other words, the same effect is secured with less throwing of the actual track by means of string lining than by using an instrument. When we add to this saving in throw the greater smoothness and regularity of line obtained by having a stake at every joint in place of every 50 or 100 ft., we begin to realize the true value of the method. Every one who is familiar with practical track work knows that if a curve is out of true alinement—that is, if it has numerous sharp and flat places—the track foreman will put in too much superelevation through the flat places of the curve and too little superelevation through the sharp places in order to secure a curve which "looks well" to his eye. The result is, naturally, rough riding track or even unsafe track. If a curve is re-alined by means of a transit, and stakes set every 50 or 100 ft. (as is customary), the foreman who lines the track is almost certain to have numbers of sharp and flat places between the stakes. It is quite true that these places will be so slightly out of line at first that their presence can scarcely be detected by the eye; but it is equally certain that these defects become greatly aggravated as time passes and the effects of high-speed trains are felt.

Determining the Throw for a Curve*†

Explanation of the Steps Entering into the Selection of the Ordinates for the Revision of the Alinement

Part 2



IF THE throws required to line a given curve are not excessive, or if it appears that, for reasons of its own, a railroad does not wish to limit the amount of throwing of the actual track, it is usually advisable to attempt to obtain a perfectly regular curve. If we take the curve used in the previous article, we shall see the method of applying the principles in order to obtain a curve which is regular throughout, without even the slight variations which characterized the solution described in the May issue. In addition to this we shall be able to find out just how much it would have been necessary to throw the track to obtain that regular curve and, therefore, what the amount of the throw would have been if the work had been done with a transit instead of with a string.

Let us assume for the purpose of this illustration that we carry straight through to the end of the curve the spiral 0, 4, 7, 11, 14, and 18, with the average revised ordinate of 20; that is, continue the curve which was started in the article in the May issue, Page 214, and then abandoned because the half-throws ran above the assigned limit at Station 19. We have the following:

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original and Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
1	0	0	0	0	0
2	5	4	1	1	0
3	7	7	0	1	1
4	9	11	— 2	— 1	2
5	9	14	— 5	— 6	1
6	16	18	— 2	— 8	— 5
7	34	20	14	6	—13
8	23	20	3	9	— 7
9	12	20	— 8	1	2
10	15	20	— 5	— 4	3
11	25	20	5	1	— 1
12	20	20	0	1	0
13	13	20	— 7	— 6	1
14	19	20	— 1	— 7	— 5
15	28	20	8	1	—12
16	25	20	5	6	—11
17	28	20	8	14	— 5
18	24	20	4	18	9
19	16	20	— 4	14	27
20	8	20	—12	2	41
21	13	20	— 7	— 5	43
22	17	20	— 3	— 8	38
23	17	20	— 3	—11	30
24	34	20	14	3	19
25	18	20	— 2	1	22
26	13	20	— 7	— 6	23
27	10	20	—10	—16	17
28	32	20	12	— 4	1
29	18	20	— 2	— 6	— 3
30	20	20	0	— 6	— 9
31	20	20	0	— 6	—15
32	33	20	13	7	—21
33	13	20	— 7	0	—14
34	14	20	— 6	— 6	—14
35	31	20	11	5	—20
36	22	20	2	7	—15
37	17	20	— 3	4	— 8
38	17	20	— 3	1	— 4
39	16	15	1	2	— 3
40	4	10	— 6	— 4	— 1
41	8	5	3	— 1	— 5
42	2	0	2	1	— 6
43	0	0	0	1	— 5

In the column headed sum of errors, we have a total of +1, which means that the total of the original ordinates is greater than the total of the revised ordinates by 1. In the half-throw column, we have a residual half-throw of —5. We must correct the column headed sum of errors first, in order to simplify the calculations. This can easily be done by making the last ordinate of 20 a 21. We then have

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original & Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half-Throw (F)
38	17	21	— 4	0	— 4
39	16	15	1	1	— 4
40	4	10	— 6	— 5	— 3
41	8	5	3	— 2	— 8
42	2	0	2	0	—10
43	0	0	0	0	—10

We can now correct the residual half-throw to zero by picking any two ordinates whose station numbers differ by ten, substracting unity from the one having the lower station number and adding unity to the one having the higher station number, in accordance with the rule that when the final half-

*This is the fifth of a series of six articles on the string lining of curves, describing the manner in which the line can be corrected by the track men without the use of instruments other than a piece of string and an ordinary rule. The first article of this series, which appeared in the January issue, Page 4, presented the merits of this practice in contrast with the use of a transit. The second article, which was published in the February issue, Page 62, described the methods of taking the measurements. The third article, which appeared in the April issue, Page 168, presented the basic principles underlying this method of determining curve alinement. The fourth article, which appeared in the May issue, Page 212, discussed the method of selecting the revised ordinates for a curve to give the minimum throw. The sixth and concluding article, which will appear in the July issue, will describe the manner of placing the stakes preliminary to the actual lining of the curve.

throw is negative we add to the ordinates near the bottom of the page and subtract from those near the top of the page. In order to reduce the work of making the change, let us pick the ordinates as close to the bottom as we can conveniently. We shall select, then the two ordinates at Stations 29 and 39, adding to the one at 39 and substracting from the one at 29. We have

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original and Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
29	18	19	— 1	— 5	— 3
30	20	20	0	— 5	— 8
31	20	20	0	— 5	—13
32	33	20	13	8	—18
33	13	20	— 7	1	—10
34	14	20	— 6	— 5	— 9
35	31	20	11	6	—14
36	22	20	2	8	— 8
37	17	20	— 3	5	0
38	17	21	— 4	1	5
39	16	16	0	1	6
40	4	10	— 6	— 5	7
41	8	5	3	— 2	2
42	2	0	2	0	0
43	0	0	0	0	0

The curve is now lined. In order to avoid making any more 21s out of the 20s, and because the 20 at Station 38 had previously been changed to 21, thus making it advisable to increase one of the ordinates of the spiral to keep the change between ordinates in keeping with that between other terms of the spiral, we increased the 15 of the spiral to a 16. As the station number of this ordinate was 39, we were compelled to pick Station 29 for the other station whose ordinate was to be changed.

An Important Point

In connection with the method of changing the final half-throw back to zero, the reader will note, if he examines the method closely, that at each succeeding station after the one whereat the first change was made, the half-throw is less by the difference in station numbers from the point of change than it was for the curve which had the residual half-throw. This is true for any curve, whether it is lined or not. That is, if we have a curve which is, say, half-finished, and we find that a certain group of throws is exceeding the desired limit, we can arbitrarily decrease that half-throw by any given amount simply by changing one or two of the ordinates a given distance ahead of the point where the half-throw is too large. To illustrate:

Suppose that we have reached the 49th station of a curve, with a half-throw of —71, which is too high to suit our purposes; we wish to change that half-throw so as to bring it to some such figure as —41. In order to do this, we have only to go back 30 stations to Station 19, and decrease the revised ordinate at that station by unity. This, also, is a valuable asset of the method.

Another valuable application of this same principle can be made when the column headed sum of errors, and the column of half-throws have the same sign at the last station. Suppose that we arrive at the last station with a sum of errors equal to +3, and a final residual half-throw of 46. We can divide 46 by 3, which gives us 2 units of 15 and one of 16. By increasing the ordinates by 2 at Station 15 back of the last station and by 1 at Station 16 back of the last station we make both columns equal to zero at the end, and the curve is lined. To make this a trifle clearer, let us suppose that the number of the last station is 53; then by adding 2 units to the revised ordinate at Station 38 (53—15) and 1 unit at Station 37 (53—16), we make the two columns referred to come out zero. Of course, the same principle applies if the sign of the last two columns is negative.

It will be noted by reference to the curve just lined that the maximum throws are now +41 and —21; the increase is small (over the first solution), and is not really indicative of the saving in throw which can generally be accomplished by the changing of a few of the ordinates by one or two. This can be more clearly realized, perhaps, if one considers that, for example, by changing by unity the tenth ordinate of a curve 93 stations long, a change of 83 in the final half-throw is effected.

Method of Compounding a Curve

In the following curve is demonstrated the method of changing the revised ordinates, and thus compounding the curve. It is usually convenient, and is a means of effecting a real saving in throw, to spiral between the two branches of the compound. As this is done merely by inserting a series between the branches, and as the operation is equally as simple as putting in all ordinates equal, one realizes almost immediately with what ease the operation can be performed. The task of realining a compound curve with an instrument (a transit) and at the same

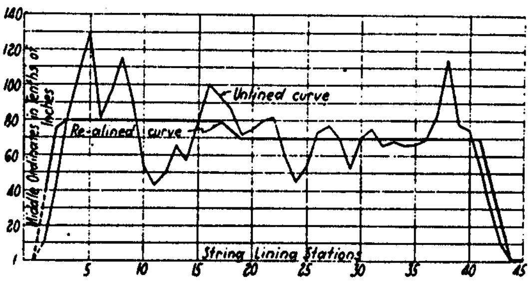


Chart of the Ordinates of a Compound Curve

time installing two spirals at each end and a spiral between the branches of the compound curves is one of great magnitude, as will be appreciated by anyone who has ever undertaken the task of doing this on an actual curve.

It might be well to state here that the track supervisor under whose jurisdiction this curve was, was entirely willing to throw the curve the large amounts required to install the revised curve. His reasons for so doing will doubtless be fully appreciated if the reader will glance at the accompanying graph of the original ordinates. It may truthfully be said that this curve can be considered as a "horrible example" of all that a good curve should not be, before it was realined. After the realining, which was made less troublesome than it would otherwise have been by the fact that the curve was relaid with new rail at the same time that the throws were made, this curve rode well.

In the case given below, the amount of change between the two branches of the compound was only 10 units, or 52.5 min. A very short spiral (0, 5, 10) was inserted, which suffices amply to effect the change. In general it may be said that the type and length of the spiral between the branches of the curves is determined by the effect of the different permissible spirals on the half-throws.

In compounding a curve the exact place at which to change the revised ordinate will always have to be determined by trial. This will not require more than two trials, as a rule, if the point first selected is anywhere close to the true point of compound.

Station Number (A)	Original Ordinate (B)	Revised Ordinate (C)	Difference Between Original and Revised Ordinates (D)	Sum of Errors to and Including Station (E)	Half Throw (F)
1	11	38	—27	—27	0
2	44	76	—32	—59	— 27
3	83	80	3	—56	— 86
4	109	80	29	—27	—142
5	130	80	50	23	—169
6	81	80	1	24	—146
7	96	80	16	40	—122
8	116	80	36	76	— 82
9	90	80	10	86	— 6
10	55	80	—25	61	80
11	43	80	—37	24	141
12	50	80	—30	— 6	165
13	66	80	—14	—20	159
14	57	80	—23	—43	139
15	79	80	— 1	—44	96
16	101	80	21	—23	52
17	94	80	14	— 9	29
18	87	75	12	3	20
19	72	70	2	5	23
20	75	70	5	10	28
21	80	70	10	20	38
22	82	70	12	32	58
23	60	70	—10	22	90
24	45	70	—25	— 3	112
25	54	70	—16	—19	109
26	74	74	4	—15	90
27	78	70	8	— 7	75
28	71	70	1	— 6	68
29	53	70	—17	—23	62
30	71	70	1	—22	39
31	76	70	6	—16	17
32	66	70	— 4	—20	1
33	68	70	— 2	—22	— 19
34	66	70	— 4	—26	— 41
35	67	70	— 3	—29	— 67
36	69	70	— 1	—30	— 96
37	83	70	13	—17	—126
38	116	70	46	29	—143
39	78	70	8	37	—114
40	75	70	5	42	— 77
41	55	70	—15	27	— 35
42	31	47	—16	11	— 8
43	10	23	—13	— 2	3
44	1	0	1	— 1	1
45	1	0	1	0	0

Note: Ordinate of 11 units at Station 1 because Station 1 is at point of reverse curve, the next station on the other side being on the reverse curve.

Spirals for Operation at a Fixed Speed

In order to install a spiral which can be run over by trains operating at a certain schedule speed, it is necessary to find a relation between the spiral series and that speed. It can be proved that such a relation can be readily derived. The relation is based on the results of tests made by the American Railway Engineering Association, as given in its bulletin No. 108 for February, 1911. In this bulletin it is stated that the maximum rate at which superelevation can be attained without discomfort to passengers is about 1 1/6 in. per second. From this fact, we derive a relation to the effect that the operating speed of a string lining spiral, in miles per hour, is given by

$$V = \frac{75.65}{\sqrt{F}}$$

where V is the velocity in miles per hour and F is the factor of the string lining spiral to be used. This relation may be more conveniently written as

$$F = 432{,}975/V^2 \quad \text{or, accurately enough,}$$
$$F = 433{,}000/V^2$$

Suppose, for example, that it is required to install a spiral which can be operated by trains running at a fixed maximum schedule speed of 50 miles per hour. Substituting 50 for V in the above equation, we have F=433,000/125,000=3.464. Thus, for such a case a 3.5 spiral can be used—either of the two following

(a) 0, 3, 7, 10, 14, 17, 21, 24, 28, etc. or
(b) 0, 4, 7, 11, 14, 18, 21, 25, 28, etc.

Having written this spiral down on the slate upon which have been transcribed the ordinates of the actual curve, we proceed to line the actual curve as was done in the foregoing cases, correcting the revised ordinates and making them the proper value.

Discussion of the Spiral

A word should be said here about the properties of the string lining spiral. It can be definitely shown that, for a given circular curve, and a given speed, there is only one transit spiral which can be used in realining a track. If other spirals are tried, one must of necessity change the degree of the circular curve slightly, or else change the direction of the tangent track line, or (as is most frequently done) compound the circular curve a short distance ahead of the point of spiral to curve.

The same thing can be done with the string line spiral, and is done, as we saw in lining the curve shown in the preceding article. Moreover, the variety of spirals which can be used in string lining far exceeds that which is available in transit lining, because of the fact that a change of only one ordinate of a revised curve in effect makes that revised curve an entirely different curve than if the change had not been made, and, inasmuch as in most curves upon which string lining computations are made there are numbers of such changes, no matter how slight or how short (in length), a number of different spirals is admissible; a fact which may at first seem contradictory of actual conditions.

For the same reason, different spirals can be used on the two ends of the same revised circular curve; in fact, the operator will generally find it a means of effecting a considerable saving in throw to use different spirals on the two ends, because of the condition of the actual track. This brings up the question of the relation of spirals to the circular curve, to the track in general, to traffic and the direction of preponderance of traffic, and to the grade line.

Relation of Spirals to Grade Lines and to Direction of Traffic

On double or multiple track roads it will almost always be found that the spiral on the entering end of the curve—that is, at the north end of a southbound track, etc.—is appreciably longer than the spiral on the leaving end. This is as might be expected, if one analyzes the causes for such a condition. Consequently, it is generally the case that the spiral on the entering end of the *revised* curve must be longer than the spiral on the leaving end. This means that the factor of the spiral on this end must be lower than the factor of the spiral on the other end.

Because of the ease with which it is possible to line a curve on paper by this method, it is practicable to take into account the grade line in conjunction

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with realining curves. This is a feature of realining to which practically no attention is ever paid, but which merits consideration because of its influence on the riding qualities of the curve and even on the safety of the curve.

For example, if a curve happens to be situated so that the leaving end is at or near the point of intersection of an ascending grade with a level grade or a descending grade (in other words, if the grade line appears in the form of an inverted V), it is practical to make the spiral to the curve at that end shorter and sharper than it would otherwise be. The train loses a great deal of its momentum and speed by the time it has reached the top of the grade (especially on ruling and maximum grades), which makes the value of V less in the formula given above and consequently, makes for a higher value of F, the spiral factor. On the other hand, when a descending grade meets another descending grade or a level grade (such that the grade line appears as an upright letter V) a long spiral is desirable. This is especially true if the grade line (on the descending grade) is such that the locomotive engineman will apply his train brakes, because a short spiral on the leaving end of such a grade, combined with the higher superelevation required by such a shorter and sharper spiral, tends to emphasize the tendency of the train to "ride" the low rail to an undesirable or even unsafe amount. This may or may not result in a derailment but is practically certain to result in throwing the spiral out of line and making for generally bad alinement throughout that entire portion of the curve.

Start at Entering End

In connection with the making of some spirals longer than others it has been the author's experience that it is quite easy to make a long spiral on that end of the curve upon which calculations are first begun, but more difficult on the other end. Consequently, it is usually a good practice to start lining (on paper, of course) all curves from the entering end of the curve, or *with* the direction of traffic. In this way, the proper spiral is secured almost automatically. In order to do this the ordinates of the original curve should preferably be taken in the field with this end in view, so that they will not have to be written backwards when they are put on the slate. This will obviate any confusion which might arise as to which end of the curve is which, and whether the throws are written opposite the right stations or whether they have been wrongly written opposite the stations in reverse order.

Setting the Stakes— the Last Step*†

Practical Suggestions for Placing Markers to Indicate the Amount of the Throw Preliminary to Lining the Curve

IN THE previous articles of this series the method of measuring the actual curve and the means of making detailed corrections in the alinement of the curve have been set forth. After these steps have been completed it is necessary, in every case, to set some sort of stakes or marks to indicate the location and amount of all corrections. It might be thought that it would be sufficient merely to start in and throw the curve at each string-lining station by the amount indicated on the slate; but such is not the case. It will be remembered that if any point on a curve is thrown either out or in, the adjacent points on either side move half as much in the opposite direction. Hence, after one station of a curve has been corrected, the original ordinate at the next one has been changed, and it is not sufficiently accurate to measure the correction from the new position of the rail.

Another argument in favor of setting stakes is that the actual work of re-alining the curve can be done at any time after the stakes have been set. It is quite apparent that every piece of track is subject to slight movements from day to day; but if stakes are set soon after the original measurements have been taken the amount of such movement is not enough to render the results untrustworthy. However, if no stakes are set, and the work of making the detailed corrections is allowed to be deferred until several months have elapsed, it is highly probable that the movement of the track will have been sufficient to render the results useless.

The Location of the Stakes

It is customary to set stakes opposite every string-lining station.. There are some permissible exceptions to this rule, and these are discussed in further detail below. The particular place in which the mark is to be placed opposite the string-lining station is to some extent at the option of the individual. Stakes can be set at almost any convenient distance from the gage line (5/8 in. below the top of the ball of the rail) of the high rail. However, of the different possible distances, only two have found much favor with track men. These are distances of one foot and of half the gage, respectively, from the correct or revised gage line of the high rail. Because of the fact that the setting of stakes for the center of the revised curve permits the roadmaster or supervisor to see at a glance the approximate distance that the track is to be shifted at each joint or station and because of the further fact that most railway engineering work uses the center line of track as the base line, and for numerous other reasons, the author is convinced that it is highly desirable to set all stakes so that they will represent the center line of the curve *after* it has been thrown.

In order to do this, it is necessary to compute the distance between the gage line of the actual curve before it is re-alined and the center line of the re-alined curve. This distance is a function of the throw at each string-lining station or joint of the curve.

Standard gage or the distance between the gage line of the two running rails is 4 ft. 8½ in. Assuming for the time being that the gage is not widened on a curve it will then be necessary, in order to set a tack in a stake to represent the center line of the curve, to set the tack half-gage or 2 ft. 4¼ in. from the gage line of the high rail.

Now, if we assume that another stake is to be set to mark an outward throw of two inches, it is immediately obvious that the new stake must be set two inches nearer the gage line of the high rail; since, if the rail is to be shifted two inches out, the center will be shifted the same amount. Hence, in order to set a stake indicating an outward throw of two inches, the tack in the stake must be set a distance from the gage line of the high rail equal to half-gage *less* two inches, or 2 ft. 2¼ in. Conversely, if the throw is in, or toward the center of the curve, the tack will have to be set two

*This is the last of a series of six articles on the string lining of curves, describing the manner in which the line can be corrected by track men without the use of instruments other than a piece of string and an ordinary rule. The first article of this series, which appeared in the January issue, page 4, presented the merits of this practice in contrast with the use of a transit. The second article, which was published in the February issue, page 62, described the method of taking the measurements. The third article which appeared in the April issue, page 168, presented the basic principles underlying this method of determining curve alinement. The fourth artic'e which appeared in the May issue, page 212, and the fifth article, which appeared in the June issue, page 258, described the method of selecting revised ordinates for a curve, to give the minimum throw.

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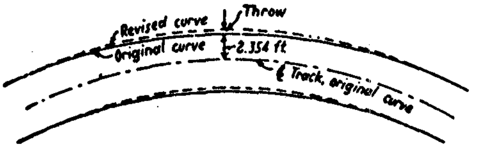
inches *farther* from the gage line of the high rail, or 2 ft. 6¼ in.

Therefore, for every inch of throw in the outward direction, the stake for the revised curve will move one inch nearer than half-gage to the gage line of the high rail of the actual curve; and for every inch throw in an inward direction, the stake will move an inch farther than the half-gage. The rule is thus formulated that the center of the revised curve is distant from the gage



The Tack Distance

The tack distance for the string-lining stake is the distance between the gage line of the high rail before re-alining and the center line of the revised curve after re-alining.

line of the high rail of the actual or original curve an amount equal to one-half the gage *minus* the throw. The minus sign takes care of the sign of the throw; if the sign is plus (out), the distance will be less; but if the sign is negative, the minus sign changes it to a plus, thus adding the throw to the half-gage distance.

If we use half-throws instead of throws, the above rule should be changed to read: The center of the revised curve is distant from the gage line of the high rail of the original curve by an amount equal to one-half the gage minus twice the half-throw. Reducing half-gage to inches, we have:

Tack distance equals 28¼ in. minus twice the half-throw.

Tables showing the tack distances for different half-throws can be made very simply. It is usually more convenient to make these tables to show feet and decimals of a foot. In order to do this, it is necessary to reduce the 28¼ in. to feet, and reduce the half-throws in eighths or tenths of inches (whichever is being used) to feet. In order to illustrate the method, the author has done this for half-throws in tenths of inches. (Note: 28¼ in. equals 2.354 ft.)

The half-throws, as obtained by the methods outlined in the preceding articles of this series, are in tenths of an inch. A half-throw of 0.1 in. means a full throw of 0.2 in. Reduced to feet, this is 2/120 or 1/60 of a foot. Each unit of half-throw, then, means that the stake is set 1/60 of a foot farther or nearer to the gage line of the original track. If we denote the half-throw in tenths of an inch by F (as in the notation used thus far), we have the relation that the distance of the tack in the stake from the gage line of the high rail on the actual unlined curve if given by

$$\text{Distance} = d = 2.354 - \frac{F}{60}$$

The author has constructed, for use in the field, table giving values of this distance, d, as computed from the above formula, for each value of the half-throw from +1 to +149, and also from —1 to —139. These tables are given herewith.

Such tables are easily constructed by differences. Thus, the difference between any two tack distances for a unit difference in throw is 1/60 of a foot, as explained above. This equals 0.1666666 ft. By subtracting 0.016 from one value, 0.017 from the next and making a correction opposite every fourth value (that is, employing the values in the order, 0.017, 0.016, 0.017, 0.017, etc.) we are enabled to write down these values very quickly. Again, every time the half-throw reaches a multiple of 6, the track distance increases or decreases, as the case may be, by 0.1 ft., so that after having computed the first six values of the tack distances for throws out and throws in, the remainder of the table can be constructed simply by

Table Giving Distance in Feet of Tack From High Rail for Half-Throws in (—) in Tenths of Inches

0	0	1	2	3	4	5	6	7	8	9
0	2.354	2.371	2.387	2.404	2.421	2.437	2.454	2.471	2.487	2.504
1	2.521	2.537	2.554	2.571	2.587	2.604	2.621	2.637	2.654	2.671
2	2.687	2.704	2.721	2.737	2.754	2.771	2.787	2.804	2.821	2.837
3	2.854	2.871	2.887	2.904	2.921	2.937	2.954	2.971	2.987	3.004
4	3.021	3.037	3.054	3.071	3.087	3.104	3.121	3.137	3.154	3.171
5	3.187	3.204	3.221	3.237	3.254	3.271	3.287	3.304	3.321	3.337
6	3,354	3.371	3.387	3.404	3.421	3.437	3.454	3.471	3.487	3.504
7	3.521	3.537	3.554	3.571	3.587	3.604	3.621	3.637	3.654	3.671
8	3.687	3.704	3.721	3.737	3.754	3.771	3.787	3.804	3.821	3.837
9	3.854	3.871	3.887	3.904	3.921	3.937	3.954	3.971	3.987	4.004
10	4,021	4.037	4.054	4.071	4.087	4.104	4.121	4.137	4.154	4.171
11	4.187	4.204	4.221	4.237	4.254	4.271	4.287	4.304	4,321	4.337
12	4.354	4.371	4.387	4.404	4.421	4.437	4.454	4.471	4.487	4.504
13	4.521	4.537	4.554	4.571	4.587	4.604	4.621	4.637	4.654	4.671
14	4.687	4.704	4.721	4.737	4.754	4.771	4.787	4.804	4.821	4.837

To use this table read the distance of the tack from the gage line of the high rail in feet and thousandths of feet opposite the proper tens unit in the vertical column at the extreme left of the page and under the unit number across the top of the page. Thus 44, or a half-throw of 4.4 inches *in*, corresponds to a tack distance of 3.087, which is read on the horizontal line opposite the figure 4 and under the vertical column headed by the the figure 4.

Table Giving Distance in Feet of Tack From High Rail for Half-Throws Out (+) in Tenths of Inches

0	0	1	2	3	4	5	6	7	8	9
0	2.354	2.337	2.321	2.304	2.287	2.271	2.254	2.237	2.221	2.204
1	2.187	2.171	2.154	2.137	2.121	2.104	2.087	2.071	2.054	2,037
2	2.021	2.004	1.987	1.971	1.954	1.937	1.921	1.904	1.887	1.871
3	1.854	1.837	1.821	1.804	1.787	1.771	1.754	1.737	1.721	1.704
4	1.687	1.671	1.654	1.637	1.621	1.604	1.587	1.571	1.554	1.537
5	1.521	1.504	1.487	1.471	1.454	1.437	1.421	1.404	1.387	1.371
6	1.354	1.337	1.321	1.304	1.287	1.271	1.254	1.237	1.221	1.204
7	1.187	1.171	1.154	1.137	1.121	1.104	1.087	1.071	1.054	1.037
8	1.021	1.004	0.987	0.971	0.954	0.937	0.921	0.904	0.887	0.871
9	0.854	0.837	0.821	0.804	0.787	0.771	0.754	0.737	0.721	0.704
10	0.687	0.671	0.654	0.637	0.621	0.604	0.587	0.571	0.554	0.537
11	0.521	0.504	0.487	0.471	0.454	0.437	0.421	0.404	0.387	0.371
12	0.354	0.337	0.321	0.304	0.287	0.271	0.254	0.237	0.221	0.204
13	0.187	0.171	0.154	0.137	0.121	0.104	0.087	0.071	0.054	0.037

To use this table, read the distance of the tack from the gage line of the high rail in feet and thousandths of feet opposite the proper tens unit in the vertical column at the extreme left of the page and under the unit number across the top of the page. Thus 44, or a half throw of 4.4 inches *out*, corresponds to a tack distance of 1.621, which is read on the horizontal line opposite the figure 4 and under the vertical column headed by the figure 4.

adding or subtracting 0.1 ft., and using the remainder of the figures as they were in the first six values. Such a table will be found of immense value to any one who lines more than one curve, inasmuch as the savings in time and labor in computing the tack distances are tremendous.

Stakes and Equipment for Driving Them

The best type of stake for string-lining purposes has proved to be a stout wooden stake of suitable material such as oak, about 1¼ in. square on top and with ready-pointed bottom. A long, slender point helps the driving and speeds up the work of getting the stake down to a firm setting. A good length for the stakes is about 18 to 30 in. depending upon the type of ballast into which they are to be driven. If the ballast is cinders, the 30-in. stake is preferable because shorter stakes are too easily moved by the track gang when the ties are lined over for the new curve. If the ballast is rock the 18-in. stake is usually quite satisfactory, unless the ballast is quite new, in which case a longer stake should be used. For such materials as chats or screenings, an intermediate length of stake is best.

In rock ballast, or in very compact ballast of any kind, a long steel bar or pin, flat on one end and with a long tapering, hardened point on the other end, will be found to be a valuable aid in speeding up the work of getting the stakes in, as such ballast frequently splits a wooden stake before it reaches sufficient depth to insure firm penetration. An old lining bar, properly hardened at the tip (which should first be beaten out straight), and cut down to a length of about 38 or 40 in., makes an admirable tool for this purpose. The pin should be driven into the rock ballast first, and as it is drawn up out of the hole, the wooden stake should be inserted quickly to prevent loose rock and dirt from falling into the hole. A steel maul, with flat ends on both sides of the head, and weighing about eight pounds, will be found most satisfactory for the work of driving both the pin and the stakes.

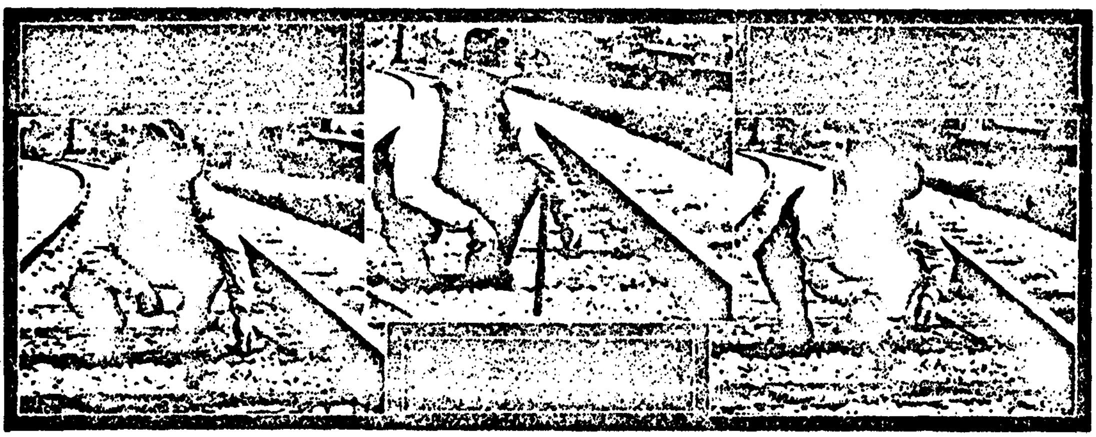
The pin should first be located by measuring from the gage of the high rail the tack distance, as shown below. Then the pin is driven in, followed by the stake. The stake should be driven quite low—so that it does not protrude more than 1½ in. above the top of the ties on each side—to prevent its being hit and broken or knocked down by the steam and air lines of passing trains, or by dragging brake beams, etc. This is especially true of those stakes which, because of small throws, are near the center of the unlined track, since such stakes are more directly in the line of the air and steam lines than those at one side.

At points where the throws are quite small, and all in the same direction—that is, either all out or all in for a distance of four or five rail lengths—some of the stakes can be omitted and the throw marked on the tie. In places where there is no throw (there are frequently such places on nearly all curves) it will suffice to write "O. K.—No Throw" on the nearest tie in yellow chalk or keel.

In setting center line stakes, the measurement from the gage line of the high rail to the tack in the stake should be made in the same manner as the original measurement was made when recording the data on the curve. In other words, if the tape was held about on a level with the tops of the two rails in taking the original ordinates before throwing, it should be held at about the same angle when the stakes are set. It goes without saying that, unless some precaution is taken to get approximately the same angle every time, the angle of measurement will be different on every stake set. As this is unsatisfactory, it is best to adopt some rule for the measuring, and then follow the rule as nearly as possible. In other words, all measurements should be made level (for example), or else all measurements should be made in the plane of the two rails. In either case, it will be necessary to sight down over the tape to the top of the stake, in order to locate the exact point at which to drive the tack. This can best be done by the aid of a small ruler or a pencil, as the eye can better determine whether the ruler or pencil is vertical than it can whether the line of sight alone is vertically over the tape graduation.

In driving the stake, the person driving it should stand near the center of the track and two feet or so away from the stake. In other words, he should stand in such a manner that he can see whether the stake is vertical and has not shifted across the track (between the two rails) as a small amount of shift this way is much more important than a greater amount along the center line of the track. Where there is any considerable amount of superelevation in the high rail, it will be found that there is a decided tendency for a stake to "slide" down-hill or toward the lower rail as it is being driven. In view of this fact, it is generally wise to set the point of the stake from a half inch to as much as three inches nearer the high rail than the actual point where it must finally be placed; after which it can be allowed to slide back to the desired point as it is being driven down.

Cases will frequently be encountered where the string-lining station or the joint is on a tie or is located between two ties where there is scarcely sufficient space in which to drive a stake. In this case, of course, it is necessary to set the stake to one side or the other. However, it should be remembered that this will change the effective length of the chord; and as the middle ordinates vary as the square of the chord length, some slight correction may be necessary where the ordinate is unusually large. For example: suppose that a curve has been measured with a 66-ft. chord, that the revised middle ordinate is found to be 56 (tenths of an inch), and that a stake cannot be set opposite a particular station, but must be set two feet farther away (toward the next station or joint). Then the actual chord length is not 66 ft., but 68 ft.; and the ordinates will vary as 66 squared is to 68 squared or as 4,356



Practical Hints for Setting Stakes

Wrong way to set the tack, the rule is at an angle — The pin used in rock ballast must be set carefully — Right way to set the tack, using pencil with rule horizontal is to 4,624 or 94.3 per cent. Consequently, 56, the ordinate to set is only 94.3 per cent of the ordinate that must be measured on account of the lengthening the chord; and, instead of measuring 56, the recorder or person driving the stakes, must measure 59 (tenths of an inch). By the same reasoning, if the next stake he drives is in the correct position, the chord length will be 2 ft. *less* (because the other one was 2 ft. longer) than 66 ft. or 64 ft.; and he must measure an ordinate of 94.0 per cent of 56 or 53. The correction for a move of 1 ft. either side of the joint is, for a chord of 66 ft., approximately 3 per cent; for a 2-ft. move and a 66-ft. chord, it is approximately 6 per cent. As even 6 per cent of a small ordinate such as 10 (or one inch) is practically negligible, the correction is advisable only when the ordinates are rather large, as illustrated.

Some roads make it a practice to set permanent track centers either in the center of the track, or at one side. On double and multiple track roads, it is usually the custom, where such stakes are set, to place them between the tracks.

As a rule, permanent track center stakes are not really permanent, but move with the ballast. Any movement obviously destroys their value. Moreover, if set in the center of the track, they cannot remain there long without being hit by some dragging part on a train—a further argument against them.

If uniform track centers are carried throughout the length of a tangent, they must necessarily be widened through the curve. Because of the poor tapes which section foremen commonly carry, and because it is very difficult for them to measure along the radial line of the curve, it usually happens that when the track centers are thus widened, they are not uniform around the curve. Added to this, is the fact that, because of the different spirals on the ends of the curve, the throws required to line two curves will not be such as to make the track centers also uniform. Small variations one way or the other from a uniformly widened center around a curve are not particularly noticeable, anyway. The value of permanent centers is, then, seen to be doubtful. In the author's opinion, they are not worth the time and labor to install them.